Intensity

Intensity curves

- Stimulus/Response - Perception
- Input/Output - Gamma

Quantization

- Uniform quantization
- Non-uniform quantization

Dithering and halftoning

- Random dither
- Ordered dither
- Clustered dither or halftone
- Error diffusion

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Measuring Light and Color

Physics: Radiometry

- The amount of *power* per wavelength interval

  Termed *radiance,* we will often use *intensity*

Psychophysics: Photometry, later colorimetry

- The relative brightness of a light source (color or black/white) when compared to a standard candle

  Termed *luminance*

- Uniform perceptual scale

  Termed *lightness*

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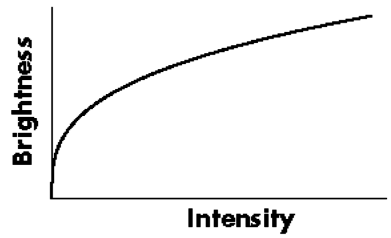
Perception of Intensities

Steven's Law:

Sensation (S) vs. Intensity (I)

$$S = I^p$$



Experiments

Sense	Exponent
Brightness	0.33
Smell	0.55
Loudness	0.60
Taste	0.80
Length	1.00
Heaviness	1.45

Stevens $B = I^{1/3}$

Weber $JND = \frac{\Delta I}{I} \approx 0.01$

Fechner $B = k \log I$

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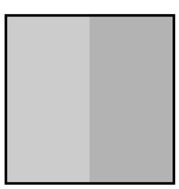
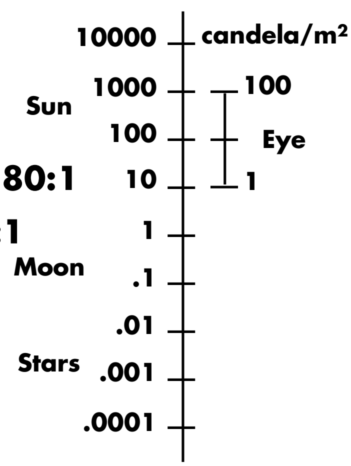
Contrast

Contrast: $\Delta I/I$

- **Human vision: 100:1**
- **Printed page: 10:1**
- **Film, photographs, monitors: 80:1**
- **Typical viewing conditions: 5:1**



10%



Adapted from Poynton 1996

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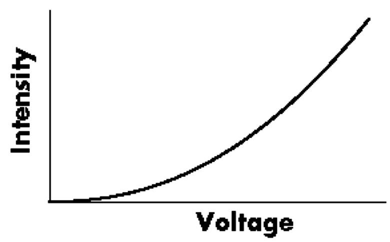
Monitor Gamma



$$\boxed{I = g \cdot (V - V_b)^{\gamma}}$$

Monitor
γ=2.5

Two knobs
Black Level (Brightness): V_b
Picture (Contrast): g

Adjustments
1st adjust to full black
Picture 0, adjust black-level
2nd adjust brightness

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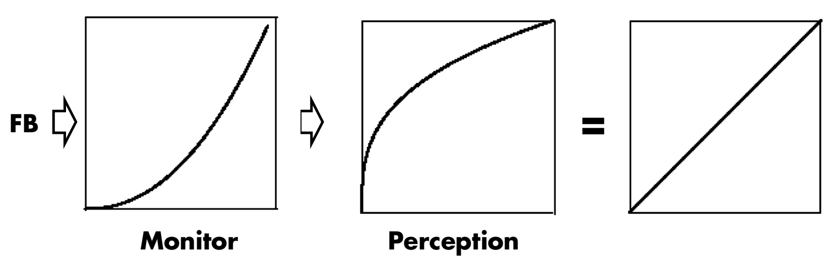
Monitor + Perception = Linear



Amazing coincidence!

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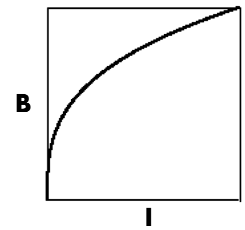
Coding in Perceptual Space

Relative brightness in intensity

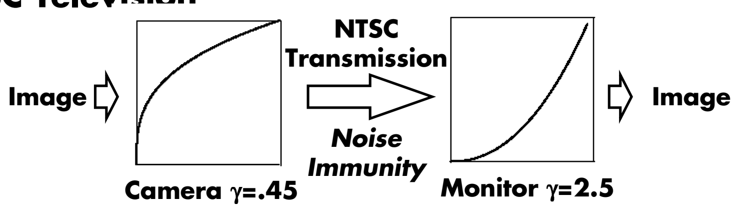
- ΔI 25 to 26 -> ΔB = 4%
- ΔI 100 to 101 -> ΔB = 1%
- ΔI 200 to 201 -> ΔB= .5%



Number of perceptual levels

log 100 / log 1.01 = 463

NTSC Television



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Perceptual vs. Intensity Space

Perceptual Space

- + good distribution of values
  - more perceivable intensities and saturated colors
- + optimal compression
  - bits used more effectively
  - less sensitivity to noise

Intensity Space

- - bad distribution of values
  - fewer perceivable intensities and colors
- + Easier to compute in
  - blending, dithering, antialiasing, lighting, ...

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Gamma Correction

Goal: Operate in linear intensity space

How: Gamma correction table

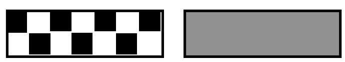
$$I = V^{\gamma}$$

$$V = P^{1/\gamma}$$

$$I = V^{\gamma} = \left(P^{1/\gamma}\right)^{\gamma} = P$$

Input	Output
0	0
1	16
2	23
3	28
4	32
...	...
...	...
251	253
252	253
253	254
254	254
255	255

Checkerboard test:



What's wrong with this table?

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Quantization

Definition: The process of turning a high precision number into a low precision number

Why?

- **Conserve memory**
- **Binary output devices**

How?

- **Minimize error (better, perceptual error)**
- **Distribute error spatially**

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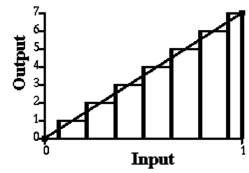
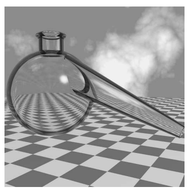
Uniform Quantization

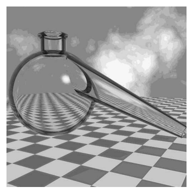
Uniform map:

$$\hat{v} = trunc((K-1)v + 0.5)$$
$$0 \le v < 1$$
$$0 \le \hat{v} < K$$

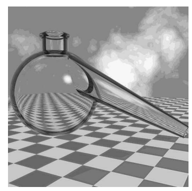




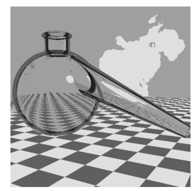
K=256



K=16



K=4



K=2

Notice contouring

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Quantization Error

Quantization introduces error

$$E^2 = \sum_{(x,y)} \left(\frac{\hat{v}(x,y)}{K-1} - v(x,y) \right)^2$$

To reduce error:

1. **Optimal quantization (minimize error)**
2. **Dithering (trade-off intensity/space error)**

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Non-uniform Quantization

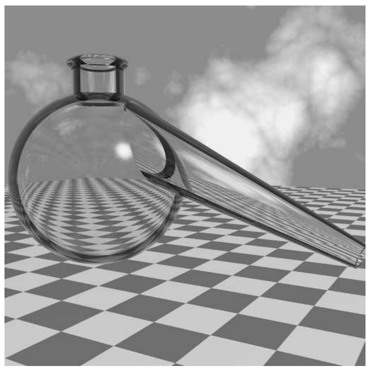
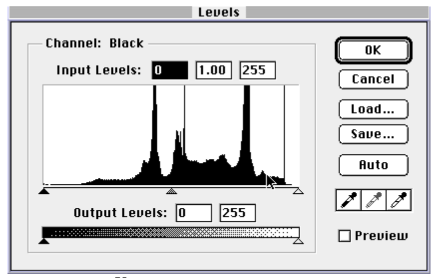


Image courtesy Don Mitchell

Histogram of Intensities



$$E^2 = \sum_{k=1}^{K} \int_{v_{k-1}}^{v_k} (v - \hat{v}_k)^2 \quad p(v) \quad dv$$

$$\hat{v}_k = \frac{v_k}{K-1}$$

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Least-Squares Optimization

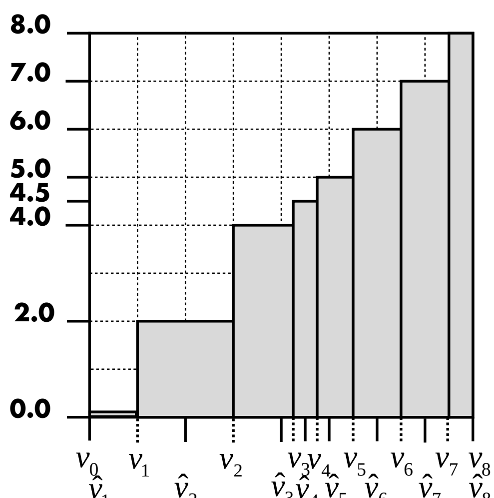
$$E^2 = \sum_{k=1}^{K} \int_{v_{k-1}}^{v_k} (v - \hat{v}_k)^2 p(v) dv$$

Least Squares

Means: $$\hat{v}_k = \frac{\int_{v_{k-1}}^{v_k} vp(v) \quad dv}{\int_{v_{k-1}}^{v_k} p(v) \quad dv}$$

Intervals: $$v_k = \frac{1}{2}(\hat{v}_{k-1} + \hat{v}_k)$$

Necessary conditions, not an algorithm



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K-Means Algorithm

1. Pick initial means: $\hat{v}_k$

2. Minimum distance classification: $v_k = \frac{1}{2}(\hat{v}_{k-1} + \hat{v}_k)$

3. Compute new means: $\hat{v}_k = \frac{\int_{v_{k-1}}^{v_k} v\,p(v)\,dv}{\int_{v_{k-1}}^{v_k} p(v)\,dv}$

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Dithering and Halftoning

Trade spatial for intensity resolution

- **Random dither; Robert's algorithm**
- **Ordered dither**
- **Clustered dither or halftones**
- **Error diffusion**

Your eye will average over an area

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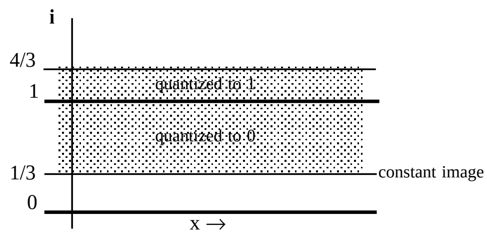
Robert's Algorithm

First add noise $\hat{v}(x,y) = trunc(K \times v(x,y) + noise(x,y))$

Then quantize $0 \le noise < 1$



Moves low freq. (average error) to high freq.

Pink(low), *Blue (high),* White(all) frequency noise

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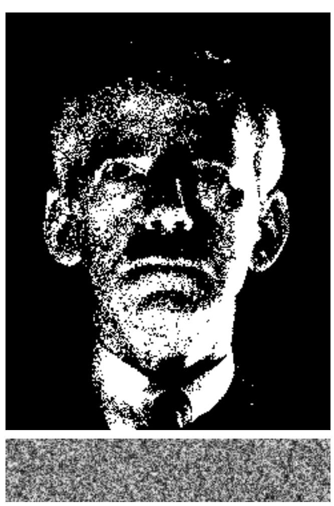
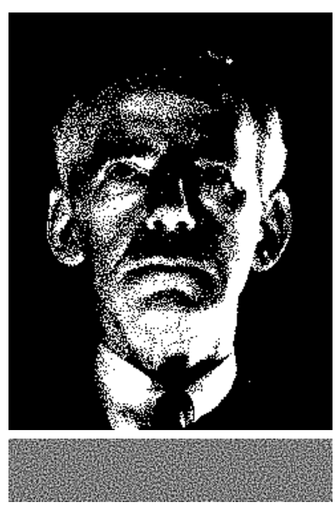
Pink vs. Blue Noise





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Noise

Noise functions

- **Some noise better than others: blue, ordered**
- **Random function vs random sequence**
- **Reproducible and order independent**
- **Precompute and store in a table (fast on the fly)**
- **Small tiled patterns sufficient**

```
for(y=0; y<ny; y++)
  for(x=0; x<nx; x++)
    vq[x][y] =
      (v[x][y]<threshold[x%M][y%M]) ? 0 : 1;
```


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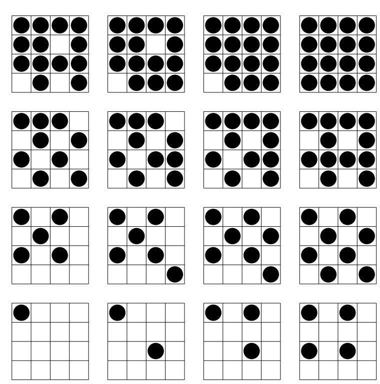
Bayer Ordered Dither Patterns

$$D_2 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}$$

$$D_n = \begin{bmatrix} 4D_{n/2} + 0 & 4D_{n/2} + 2 \\ 4D_{n/2} + 3 & 4D_{n/2} + 1 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{bmatrix}$$



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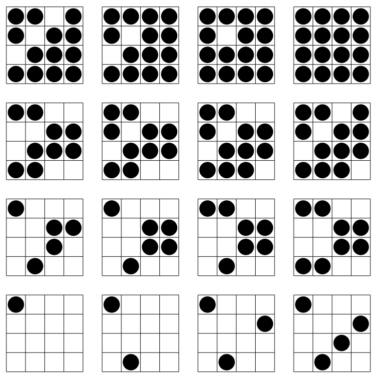
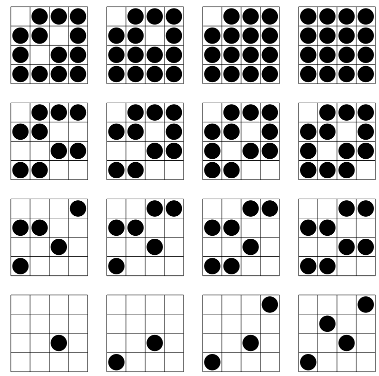
Magic Squares



$$M_4 = \begin{bmatrix} 0 & 6 & 13 & 11 \\ 9 & 15 & 4 & 2 \\ 14 & 8 & 3 & 5 \\ 7 & 1 & 10 & 12 \end{bmatrix}$$



$$P_4 = \begin{bmatrix} 15 & 8 & 5 & 2 \\ 4 & 3 & 14 & 9 \\ 10 & 13 & 0 & 7 \\ 1 & 6 & 11 & 12 \end{bmatrix}$$

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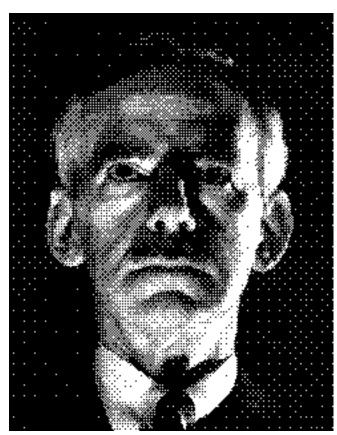
Comparison

Blue Noise



Bayer or Ordered Dither

CS248 Fall 98 Lecture 3

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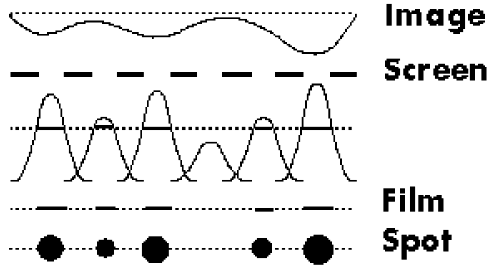
Classic Halftoning

Printers: *Binary Blobs* of ink
- Clustered pattern
- Enlarging spot

Classical printer scree
- Spot function
- Line frequency
- Tilt angle



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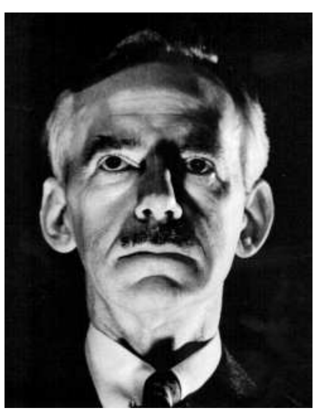
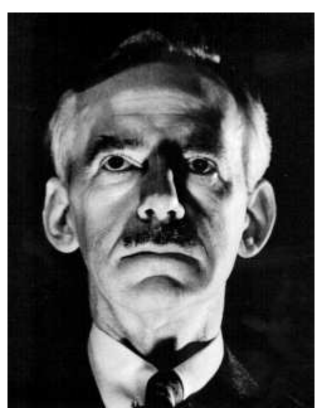
Line Screens

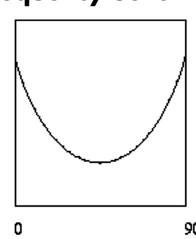


line 10 angle 0



line 10 angle 45

Frequency Sensitivity



45 degrees best

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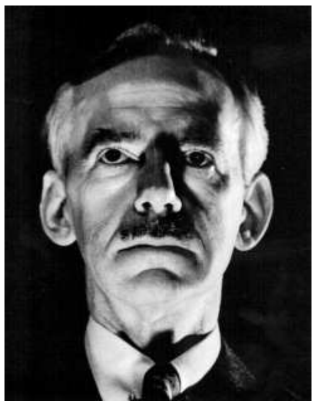
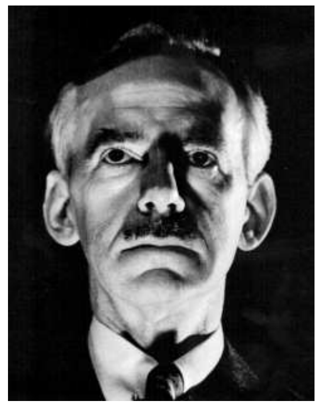
Line Screens

High spatial resolution means low intensity resolution



line 100 angle 45



line 200 angle 45

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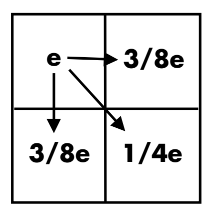
Error Diffusion

Idea: Quantize, then distribute error to neighbors

```
for(y=0; y<ny; y++)
  for(x=0; x<nx; x++){
    vq[x][y] = quantize(v[x][y]);

    e = v[x][y] - vq[x][y];

    i[x+1][y] += 3/8*e;
    i[x][y+1] += 3/8*e;
    i[x+1][y+1] += 1/4*e;
}
```



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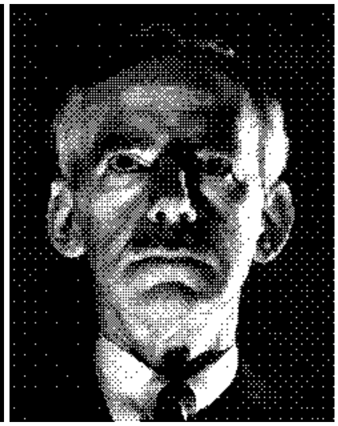
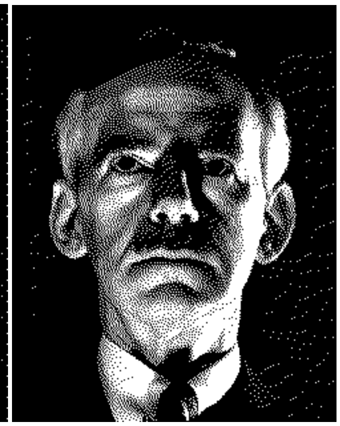
Comparison

Blue Noise



Bayer Dither



Floyd-Steinberg

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