Signal Processing and Sampling

The sampling and reconstruction process

- Real world: continuous
- Digital world: discrete

The Fourier Transform and spectra

Filtering

The convolution theorem

The sampling theorem

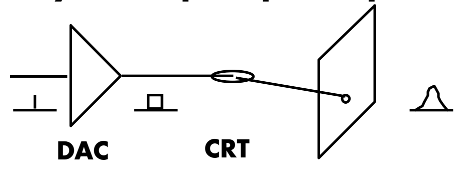
Aliasing and antialiasing

Displays = Signal Reconstruction

All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.

Examples:

- DAC's: sample and hold
- Cathode ray tube: phosphor spot and grid



Imagers = Signal Sampling

All imagers convert a continuous image to a discrete sampled image by integrating over the active area of a sensor.

$$R(i,j) = \int_{A_{i,j}} E(x,y) P_{i,j}(x,y)\, dx\, dy$$

Examples:

- **Retina: photoreceptors**
- **CCD array**
- **Vidicon: phosphors**

Virtual computer graphics cameras do not integrate, instead they simply sample ...

Sampling in Computer Graphics

Artifacts due to sampling - Aliasing

- **Jaggies**
- **Moire**
- **Flickering small objects**
- **Sparkling highlights**
- **Temporal strobing**

Preventing these artifacts - Antialiasing

First computer graphics commandment:

Thou shalt not make pictures with computational artifacts

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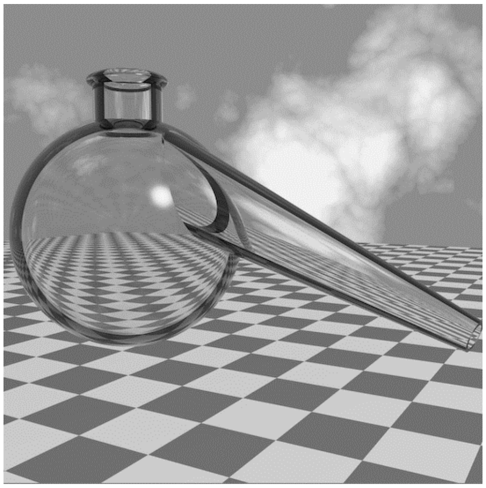
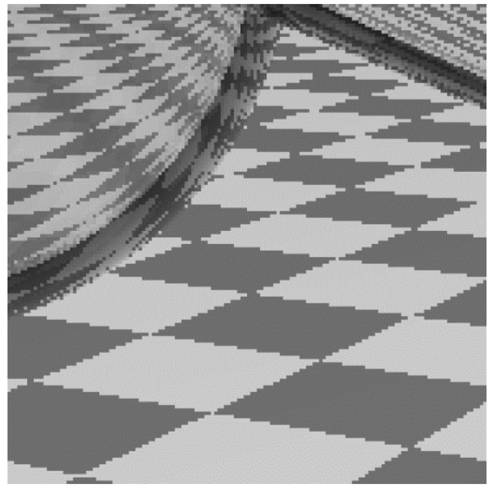
Jaggies





Staircasing or jaggies

Retort sequence by Don Mitchell

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Spectral Analysis

Spectral representation treats the function as a weighted sum of sines and cosines

Each function has two representations

- Spatial (time) domain - normal representation
- Frequency domain - spectral representation

The *Fourier transform* converts between the spatial and frequency domain

$$\Rightarrow F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx \Rightarrow$$

$$\Leftarrow f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x}\, d\omega \Leftarrow$$

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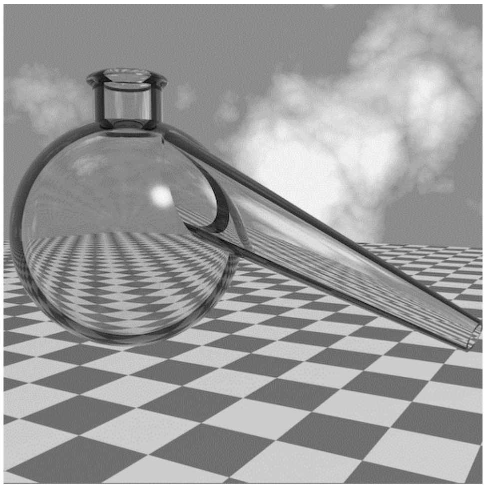
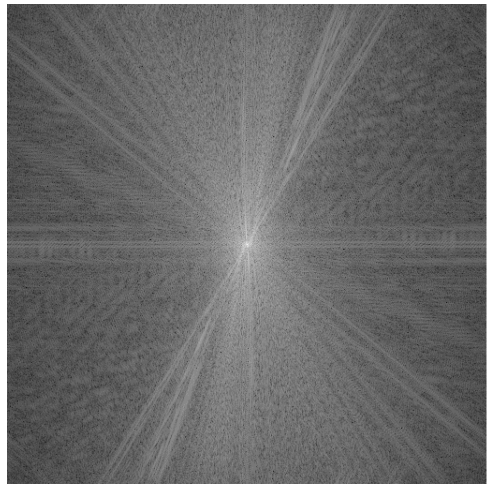
Spatial and Frequency Domain

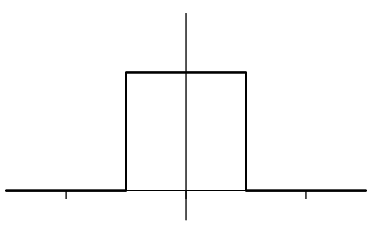


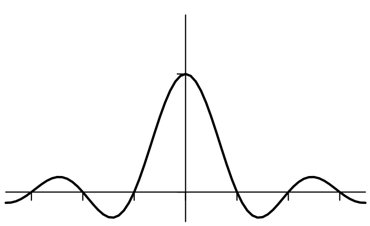
Spatial Domain



Frequency Domain

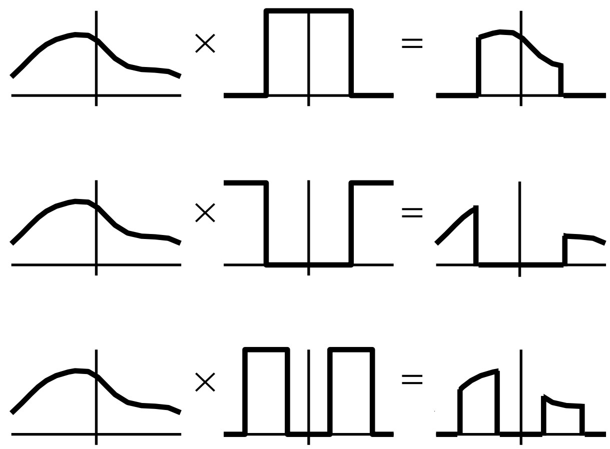
The "Sinc" Function



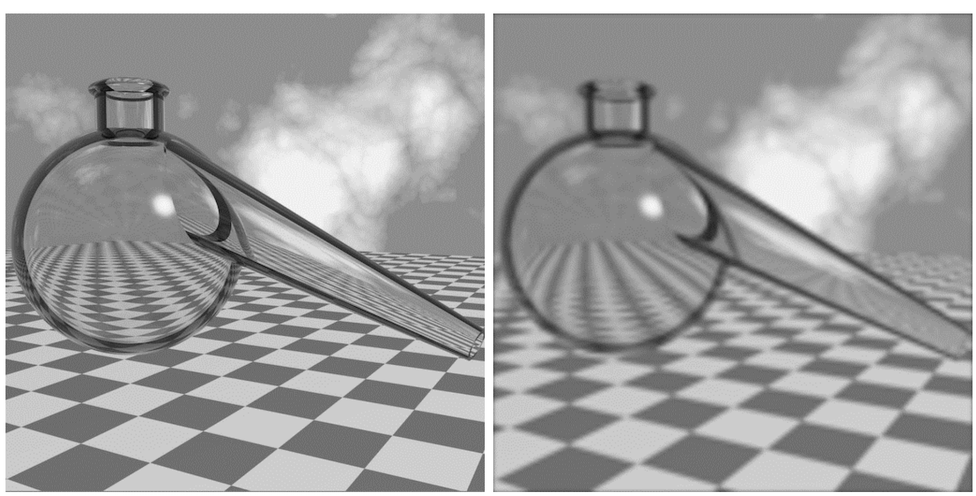


$$
\begin{aligned}
\int_{-\infty}^{\infty} square(x) e^{-iwx}\, dx &= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-iwx}\, dx \\
&= \left. \frac{e^{-iwx}}{-iwx} \right|_{-\frac{1}{2}}^{\frac{1}{2}} \\
&= \frac{e^{-\frac{1}{2}iw} - e^{\frac{1}{2}iw}}{\frac{1}{2}iw + \frac{1}{2}iw} \\
&= \frac{\sin \frac{1}{2} w}{\frac{1}{2} w} \\
&= \operatorname{sinc} f
\end{aligned}
$$

Filtering: Frequency Domain

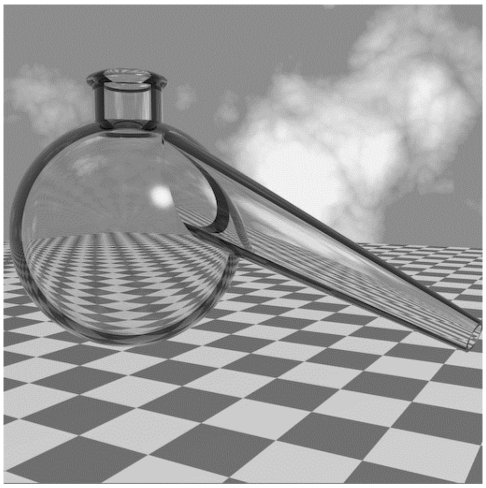


Low-Pass Filter

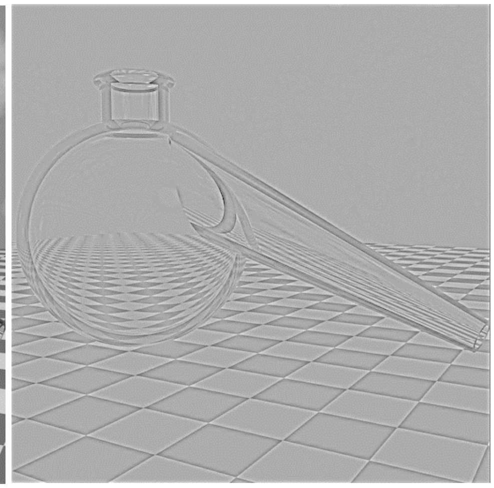


Original **Blurred**

High-Pass Filter



Original



Edge enhancement

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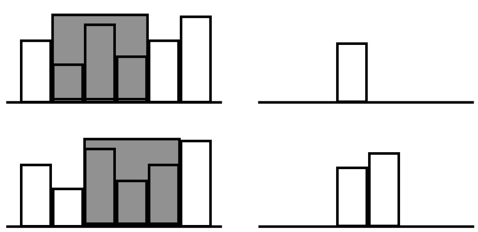
Filtering: Spatial Domain

Filtering



Convolution of two functions

$$h(x) = f \otimes g = \int f(y) g(x-y)\, dy$$

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Convolution Theorem

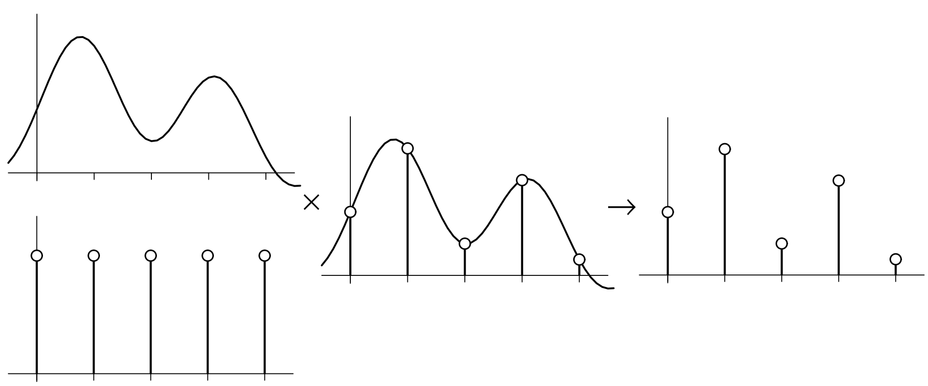
Theorem: Multiplication in the frequency domain is equivalent to convolution in the space domain.

$$F \times G \quad \Leftrightarrow \quad f \otimes g$$

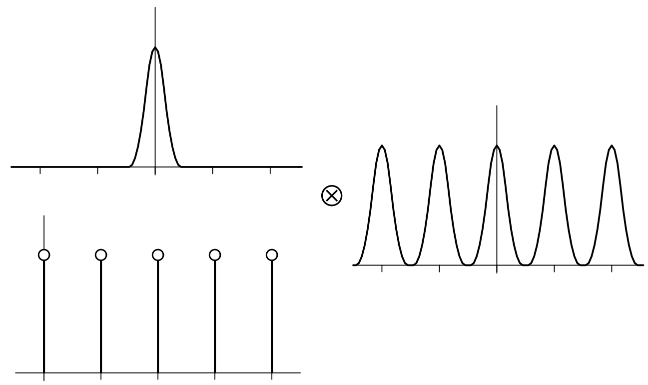
Symmetric Theorem: Multiplication in the space domain is equivalent to convolution in the frequency domain.

$$f \times g \quad \Leftrightarrow \quad F \otimes G$$

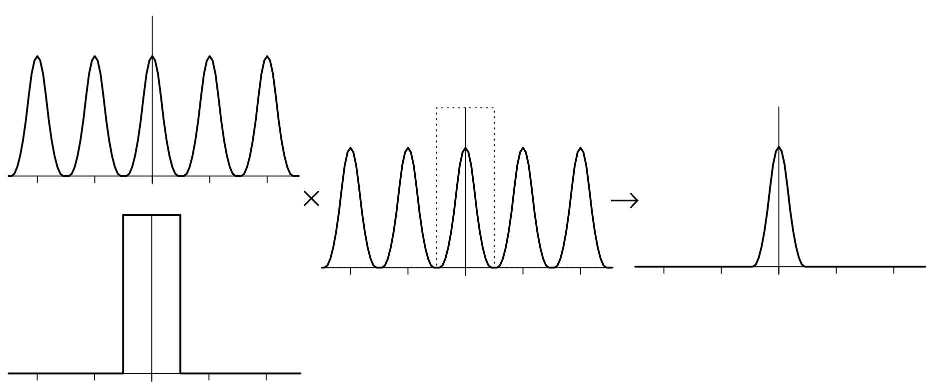
Sampling: Spatial Domain



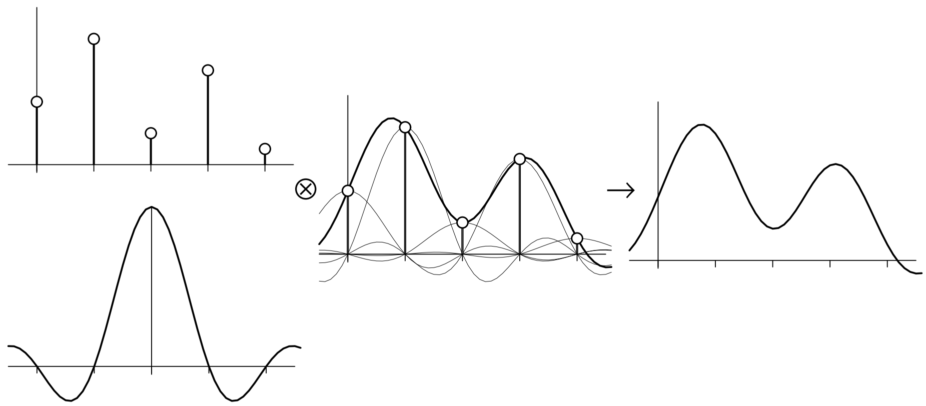
Sampling: Frequency Domain



Reconstruction: Frequency Domain



Reconstruction: Spatial Domain



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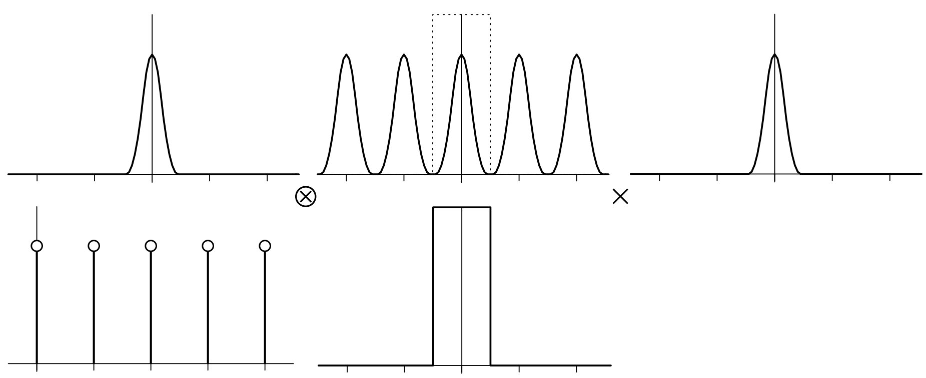
Sampling and Reconstruction



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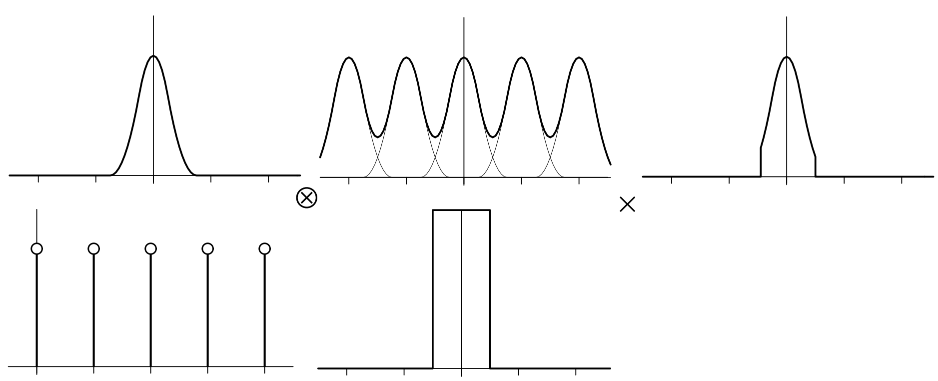
Sampling Theorem

This result if known as the *Sampling Theorem* and is due to Claude Shannon who first discovered it in 1949

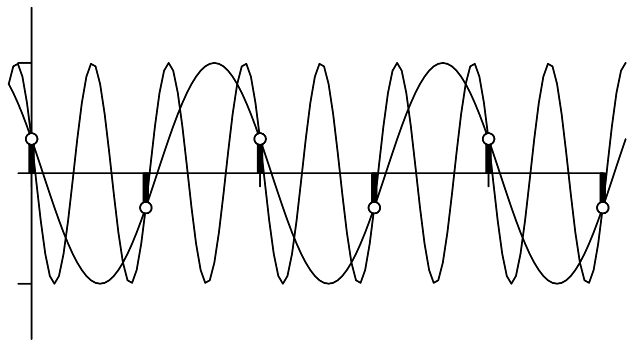
> ***A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the sampling frequency***

For a given bandlimited function, the rate at which it must be sampled is called the *Nyquist Frequency*

Undersampling: Aliasing



"Aliases"



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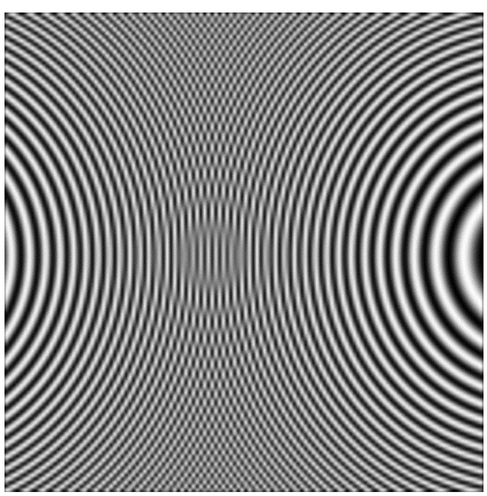
Sampling a "Zone Plate"



Note Moire Patterns

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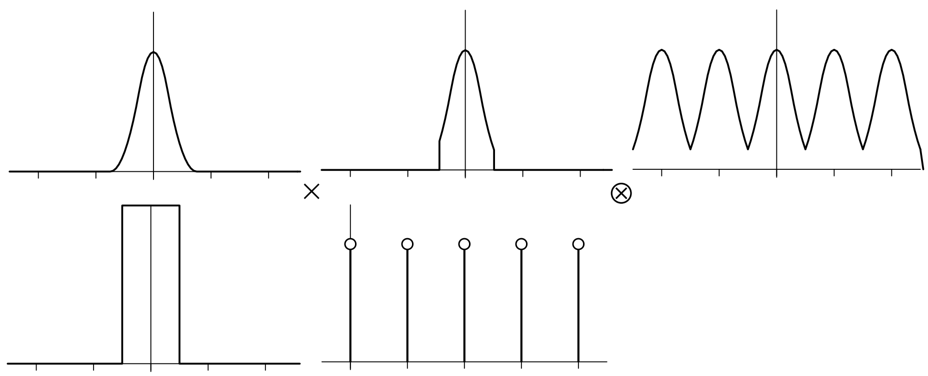
Antialiasing by Prefiltering



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Antialiasing by Prefiltering

Ideally: perfectly low-pass filter (convolve with a sinc function) to bandlimit the function to the Nyquist sampling rate.

Typically: convolve with a box filter (square pixel area average) or circular filter (circular pixel average) before sampling

Practically:

- **Constant colored lines and polygonal fragments doable**
- **Complex environments not doable**

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