Viewing Transformations

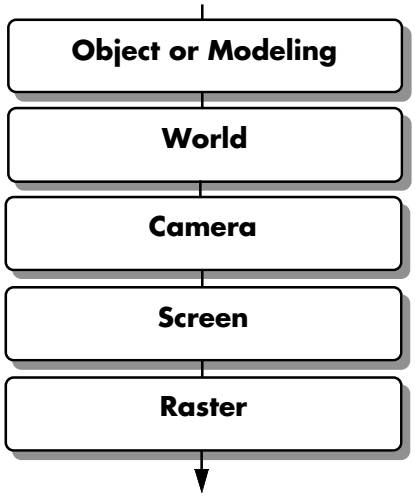
Viewing coordinate systems

Planar geometric projections

- **Orthographic**
- **Perspective**
  - **Perspective depth**
  - **Vanishing points and lines**
  - **Binocular stereo**
- **Axonometric**
- **Oblique**

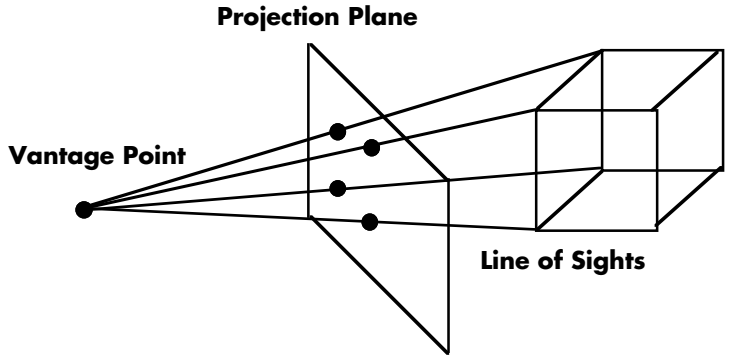
Copyright © Pat Hanrahan

Viewing Coordinate Systems



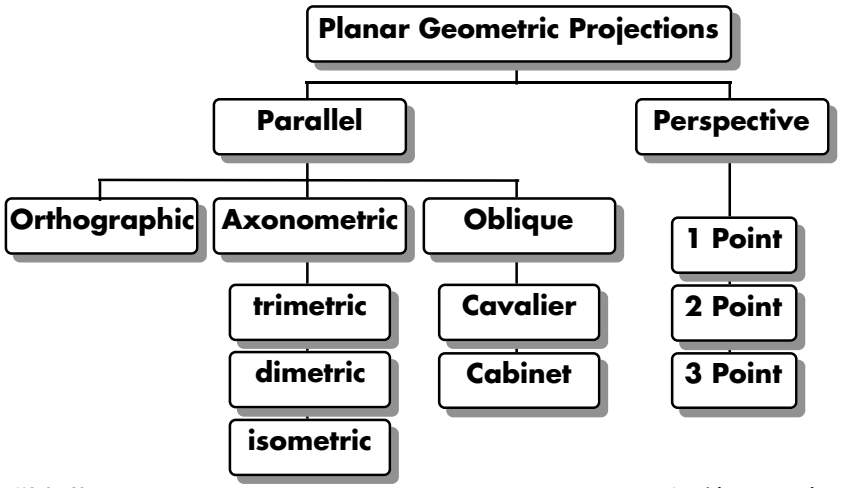
Copyright © Pat Hanrahan

Planar Geometric Projection



Copyright © Pat Hanrahan

Types of Planar Geometric Projections



Copyright © Pat Hanrahan

"Synthetic Camera"

Cameras in GL

- **Fix camera, move world relative to camera**
- **Always perform projection along the z-axis**

```
gluLookat(ex,ey,ez,atx,aty,atz,upx,upy,upx);
```

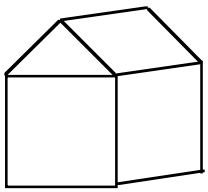
- **Similar to the "airplane" example from last lecture**

Better: Transform the camera like any other object

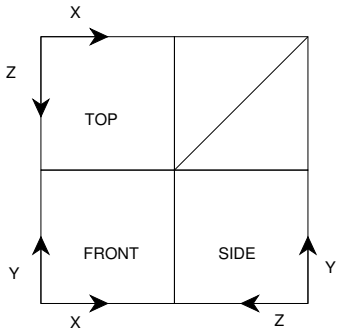
- **Then find the inverse transformation of the world wrt to the camera**

Copyright © Pat Hanrahan

Orthographic Projections



Px=S(0,1,1)
Py=S(1,0,1)
Pz=S(1,1,0)



Multiview Drawing

Copyright © Pat Hanrahan

Orthographic Transformation

`glOrtho(float l, r, b, t, n, f);`

Transforms to the unit cube: objects clipped to the cube

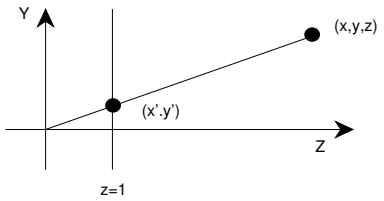
z's preserved for hidden surface calculations

Converts RHCS to LHCS: n and f relative

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Copyright © Pat Hanrahan

Perspective Projection



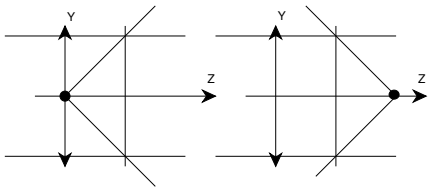
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ * & * & * & * \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Copyright © Pat Hanrahan

Perspective Depth

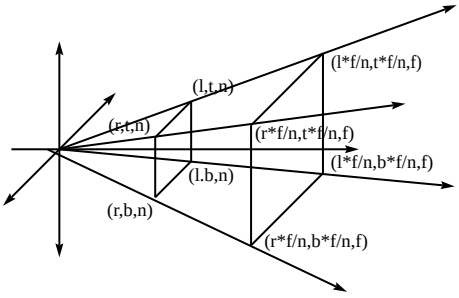
How should z transform?

$$z' = \frac{az + bw}{cz + dw}$$



Copyright © Pat Hanrahan

Perspective Frustum



$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

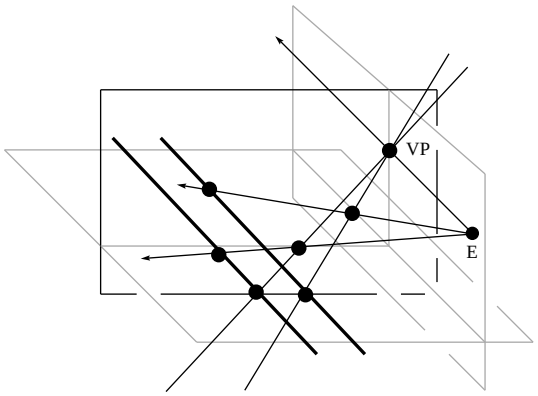
```
glFrustum(float l, r, b, t, n, f);
```

$$\lim_{f\to\infty} -\frac{f+n}{f-n} = -1$$

$$\lim_{f\to\infty} -\frac{2fn}{f-n} = -2n$$

Copyright © Pat Hanrahan

Vanishing Points and Lines



Copyright © Pat Hanrahan

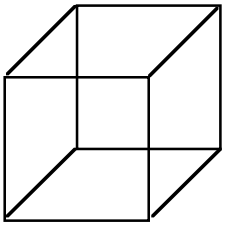
Axonometric Projection

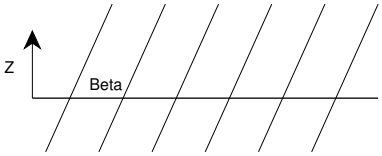
Classify parallel projections by *foreshortening* of principal directions

- **trimetric: unequal**
- **dimetric: two equal**
- **isometric: three equal**

Copyright © Pat Hanrahan

Oblique Projection





Cavalier

$$x' = x + z$$
$$y' = y + z$$

Cabinet

$$x' = x + \frac{1}{2}z$$
$$y' = y + \frac{1}{2}z$$

Copyright © Pat Hanrahan