CS 348B - Computer Graphics: Image Synthesis Techniques
Stanford University

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Spring, 1996
Handout #15

Written assignment #1 - sampling and antialiasing

Due Tuesday, April 30

(1) Draw and label a sketch of the function obtained by convolving two 2D rect functions, i.e. $\Pi(x, y) * \Pi(x, y)$. Make the sketch clear enough to convey all salient aspects of the function.

(2) The triangle function of width 2 and unit area is frequently used as a filter in computer graphics. In 1D, it is defined as:

$$\Lambda(x) = \begin{cases} 1 + x & -1 \le x < 0 \\ 1 - x & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Using the definition of convolution, prove that reconstructing a sampled 1D signal by convolving it with the triangle function $\Lambda(x)$ corresponds to linearly interpolating its samples. Asume that the samples are taken one unit apart.

(3) Suppose that we are given a continuous, integrable, real-valued function f of one real variable x. We wish to represent f over an interval $(n - 1)s$ by a set of n samples. Uniform sampling would consist of evaluating the function at n discrete points evenly spaced s units apart in x. Suppose that we implemented supersampling and averaging down by evaluating the function at nm discrete points evenly spaced s/m units apart, then computing the average of each adjacent set of m samples. Suppose also that we implemented analytic filtering by convolving f with a box filter whose non-zero width is τ and evaluating the result of this convolution at n discrete points evenly spaced s units apart. For this problem, let $\tau = s$.

Derive an expression that represents the supersampling and averaging down method described above. Derive a second expression that represents the analytic filtering method described above. Compare these two expressions in the limit as $m- > \infty$. What does this analysis suggest about the comparative image quality obtainable using each of these two techniques?

(4) In switching from a CRT, which displays pixels as Gaussian spots, to an LCD of comparable resolution, which displays pixels as 2D rects, how should one change the filter(s) employed during supersampling and averaging down? State any assumptions you make about the filters.

(Problem (5) is on the back of the page.)

(5) Consider the following four supersampling schemes. (a) and (b) and regular and non-adaptive, and (c) and (d) are stochastic and adaptive. Aside from these differences, each scheme violates one of the rules of thumb for good sampling schemes, as covered in class. Identify the violation in each scheme.

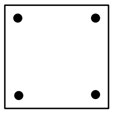
(a) 4 rays are cast per pixel using the pattern shown.

(b) 5 rays are cast per pixel using the pattern shown.

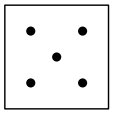
(c) 4 rays are cast per pixel, uniformly jittered within the pixel. If the pixel is complex by some metric, it is subdivided into quadrants as shown and 16 new rays are cast, 4 per quadrant, each uniformly jittered within its quadrant. The complexity test is now repeated on the 4 rays in each quadrant, recursively, down to some minimum quadrant size.

(d) 4 rays are cast per pixel, each uniformly jittered from its nominal location (solid circles) by an amount equal in size to the dotted square. If the pixel is complex by some metric, it is subdivided into quadrants as shown and 5 new rays are cast (solid circles), each uniformly jittered from its nominal location shown by an amount equal in size to the (now smaller) dotted square. The complexity test is now repeated on the 4 rays associated with each quadrant, recursively, down to some minimum quadrant size. This scheme has been proposed by many students in past CS 348B projects. It actually has two problems, one obvious and one subtle; can you find both?

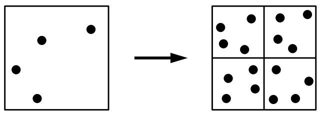
In each scheme, the solid box surrounding the pixel is intended to indicate its width relative to the interval between pixels. You may ignore the issue of how to average together the supersamples. You may assume that rays are shared among pixels or quadrants in any reasonable way; state your assumption.



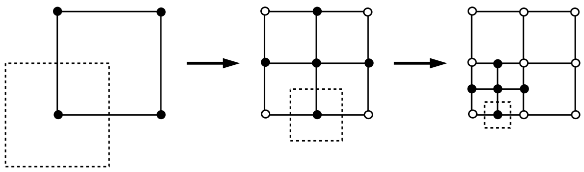
(a)



(b)



(c)



(d)

(6) **For extra credit:** Why are the scanlines in an interlaced television system like NTSC horizontal? Hint: think spatiotemporally.