Photographic optics

CS 448A, Winter 2010

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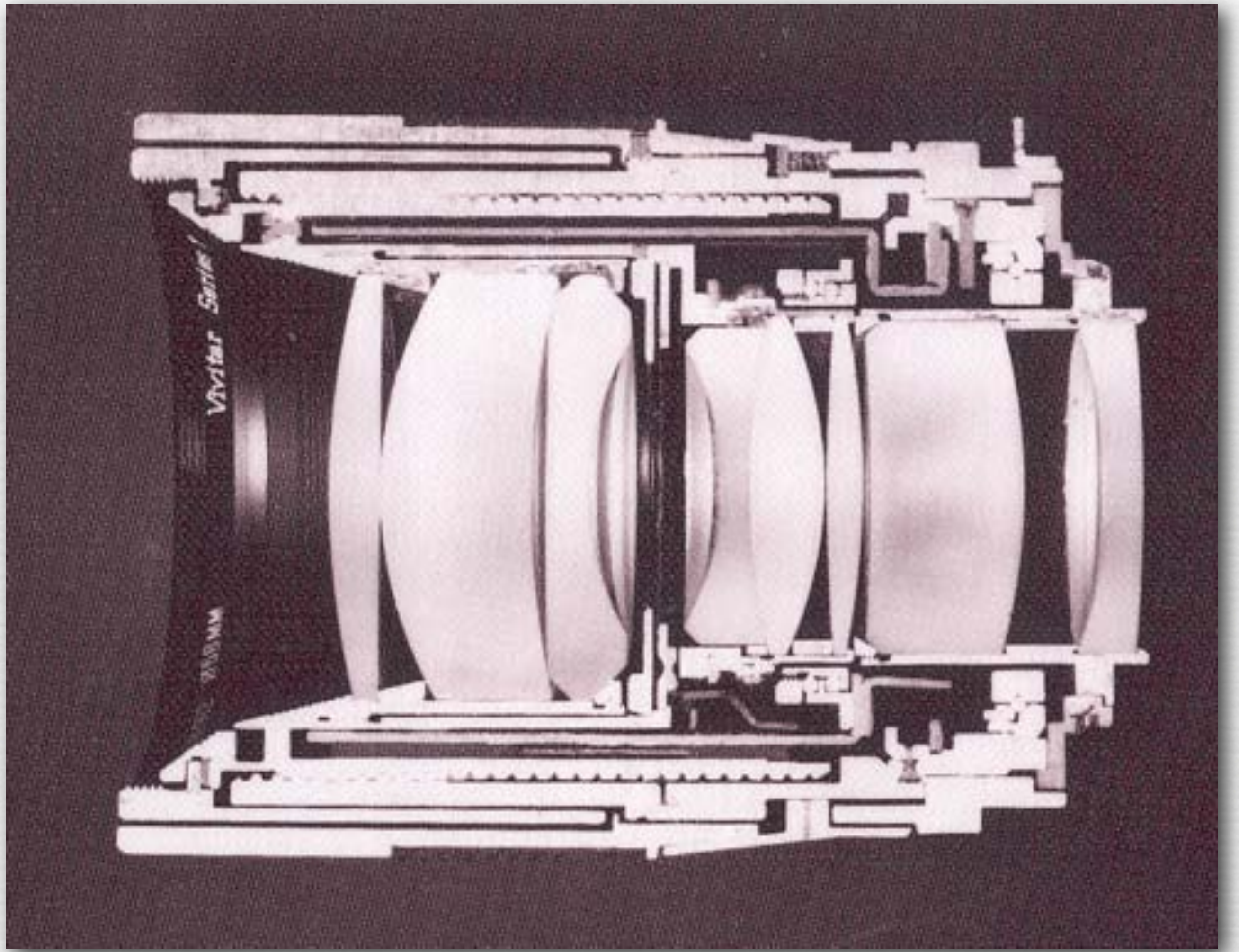
Outline

- pinhole cameras
- thin lenses
  - graphical constructions, algebraic formulae
- lenses in cameras
  - focal length, sensor size
- thick lenses
  - stops, pupils, perspective transformations
- exposure
  - aperture, shutter speed (ISO comes later)

- depth of field
- aberrations...

Cutaway view of a real lens

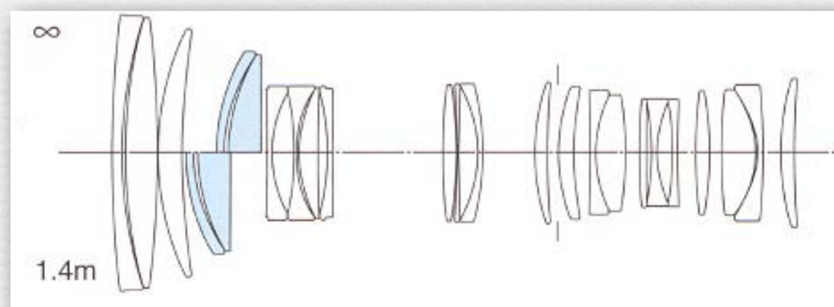


Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*

Lens quality varies

- Why is this toy so expensive?
  - EF 70-200mm f/2.8L IS USM
  - $1700

- Why is it better than this toy?
  - EF 70-300mm f/4-5.6 IS USM
  - $550



- Why is it so complicated?

(Canon)

Stanford Big Dish

Panasonic GF1

Panasonic 45-200/4-5.6
zoom, at 200mm f/4.6
$300

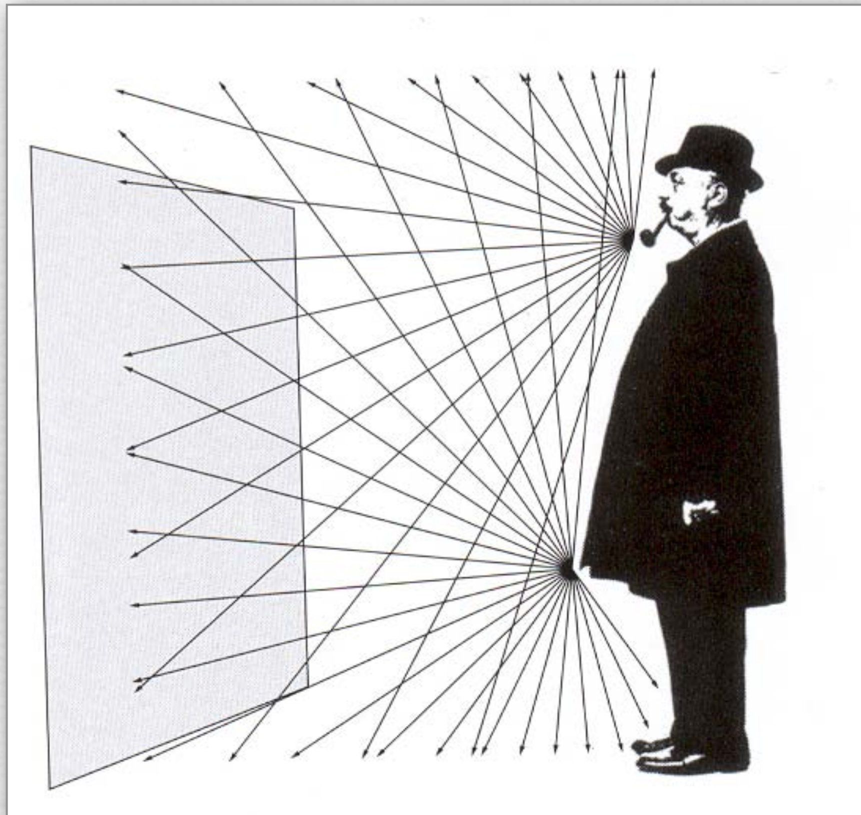
Leica 90mm/2.8 Elmarit-M
prime, at f/4
$2000

Zoom lens versus prime lens

Canon 100-400mm/4.5-5.6 zoom, at 300mm and f/5.6
$1600

Canon 300mm/2.8 prime, at f/5.6
$4300

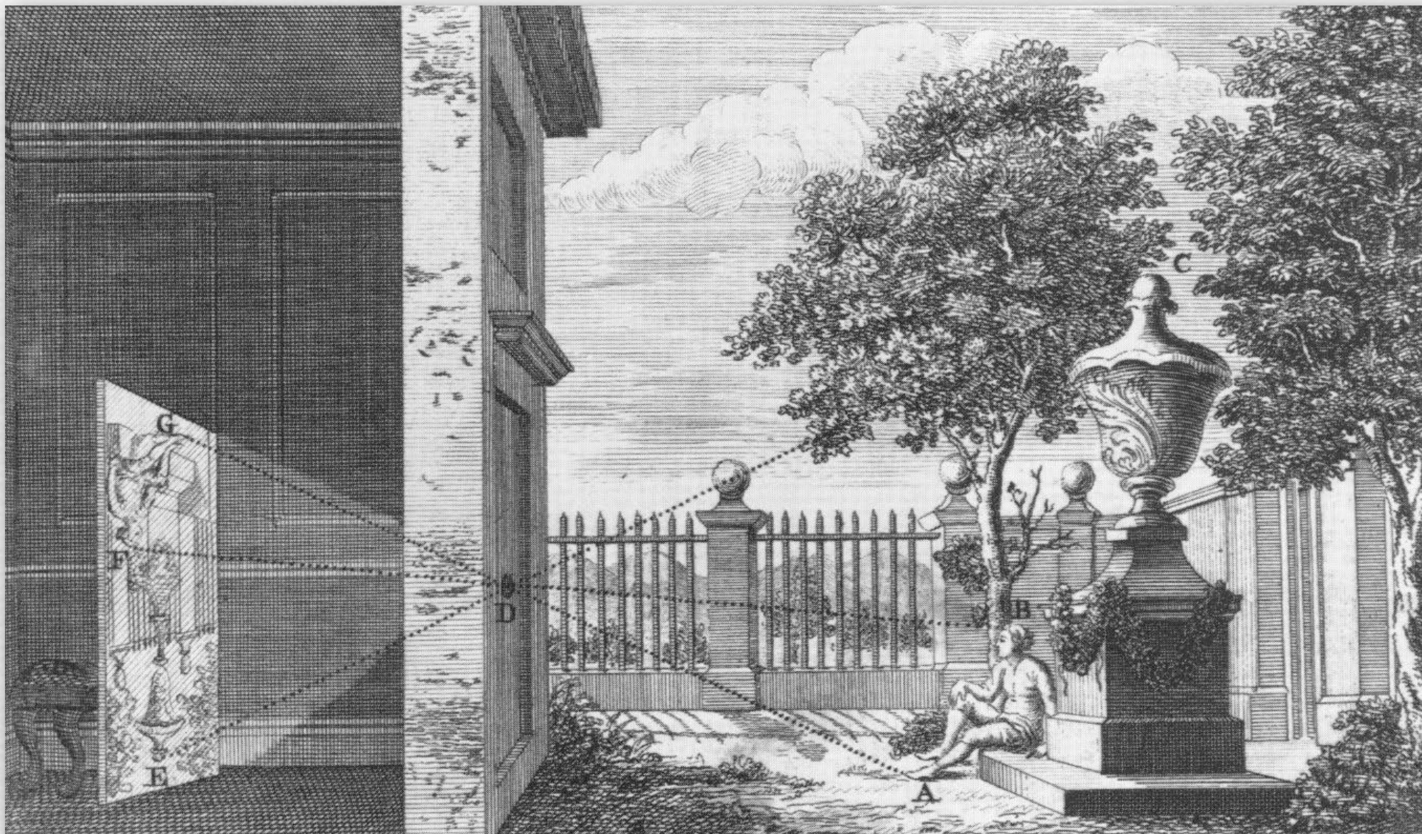
Why not use sensors without optics?



(London)

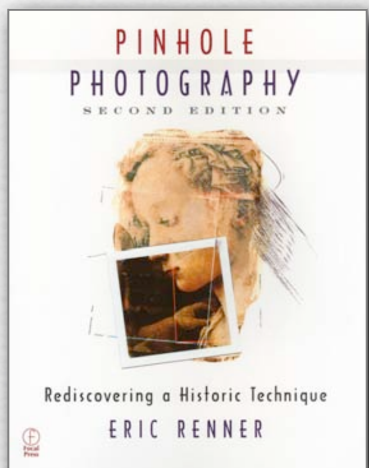
- each point on sensor would record the integral of light arriving from every point on subject
- all sensor points would record similar colors

Pinhole camera
(a.k.a. *camera obscura*)



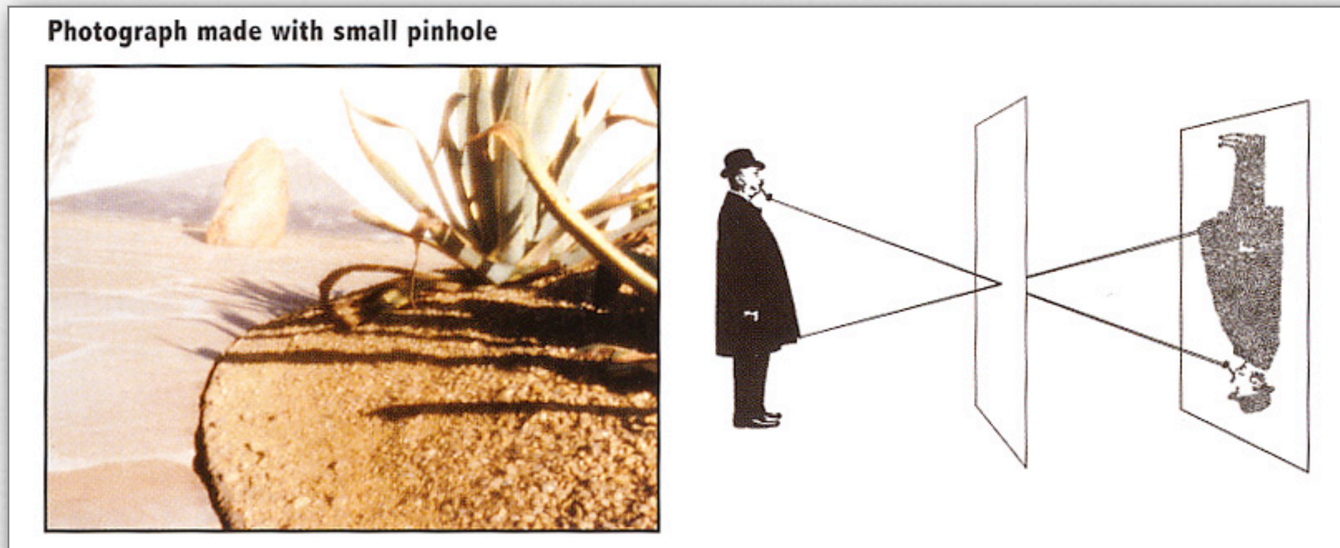
Pinhole photography

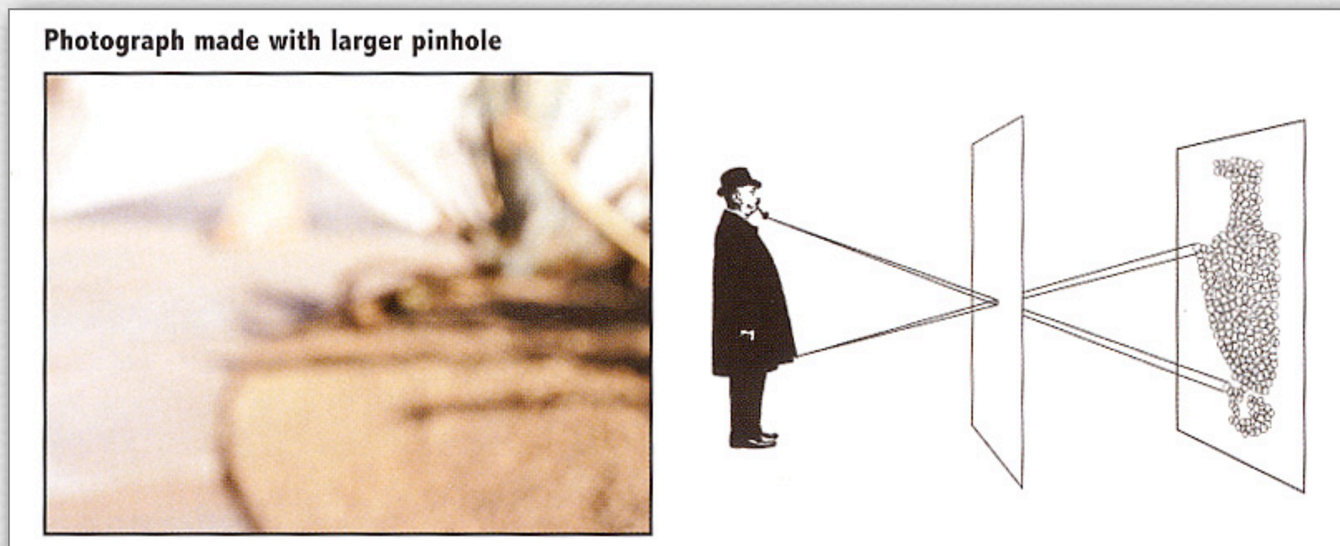
- no distortion
  - straight lines remain straight
- infinite depth of field
  - everything is in focus



(Bami Adedoyin)

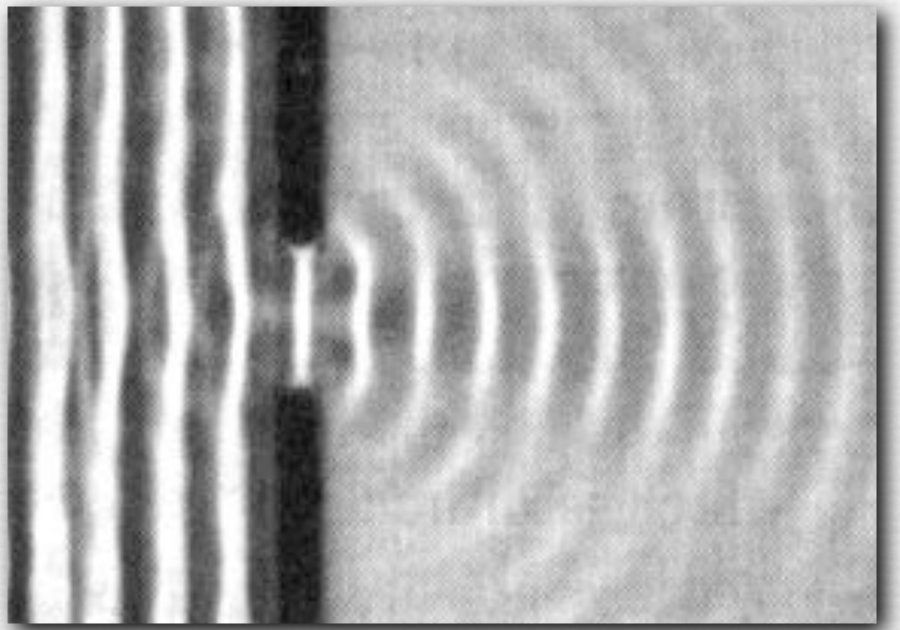
Large pinhole causes geometric blur





(London)

Small pinhole causes diffraction blur

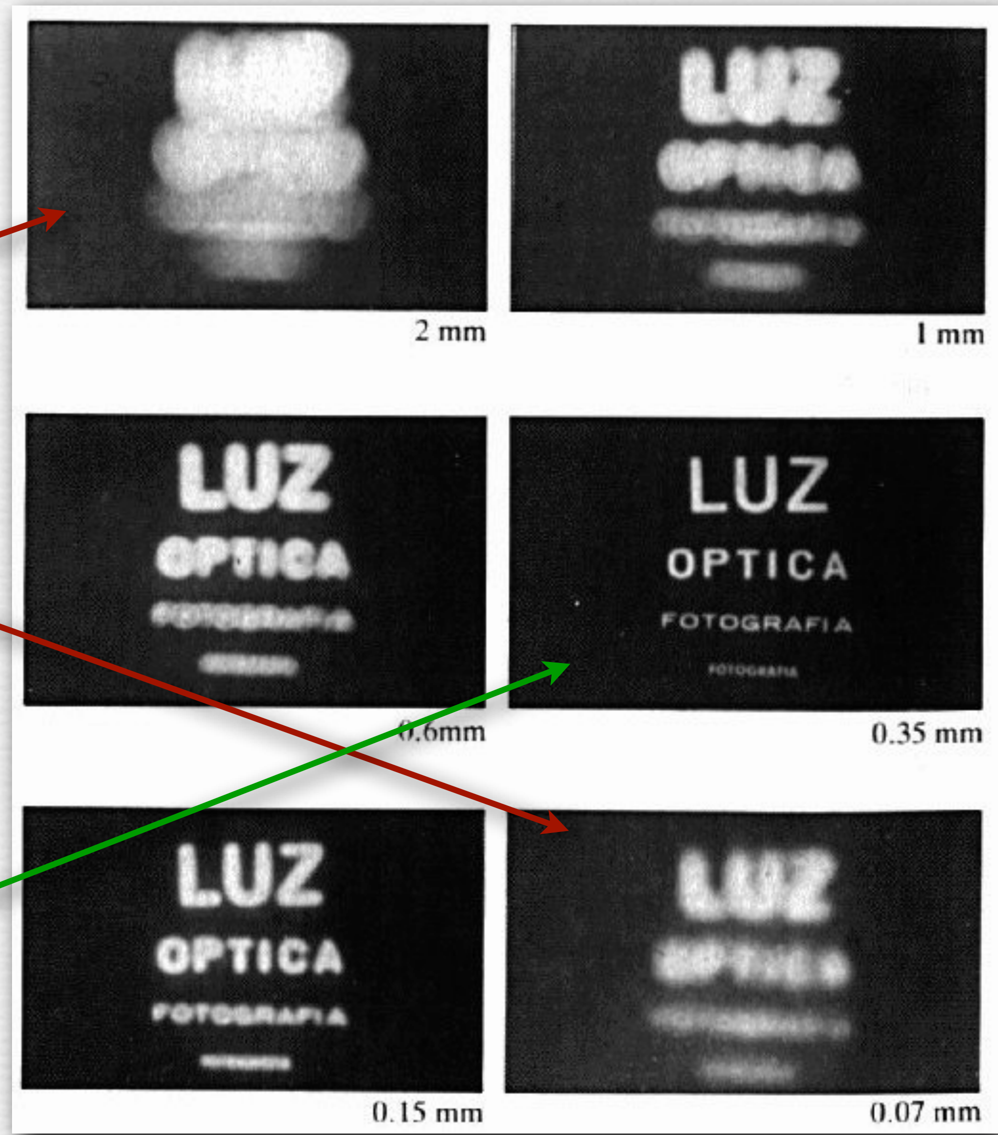


(Hecht)

- smaller aperture means more diffraction
- due to wave nature of light

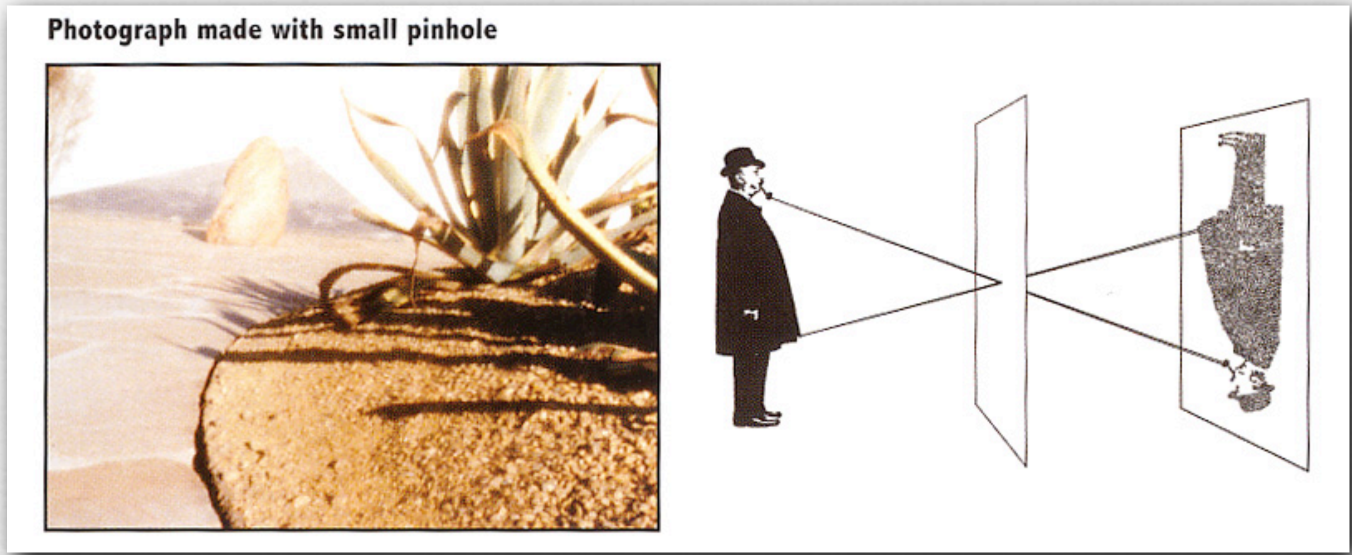
Examples

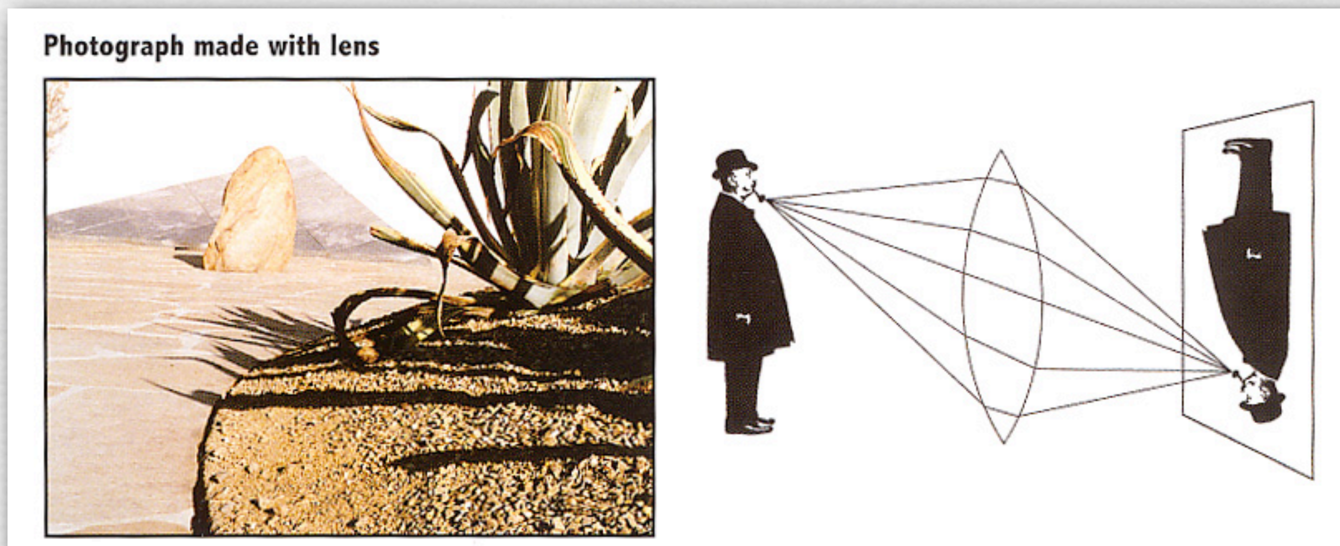
- large pinhole
  → geometric blur
- small pinhole
  → diffraction blur
- optimal pinhole
  → very little light



(Hecht)

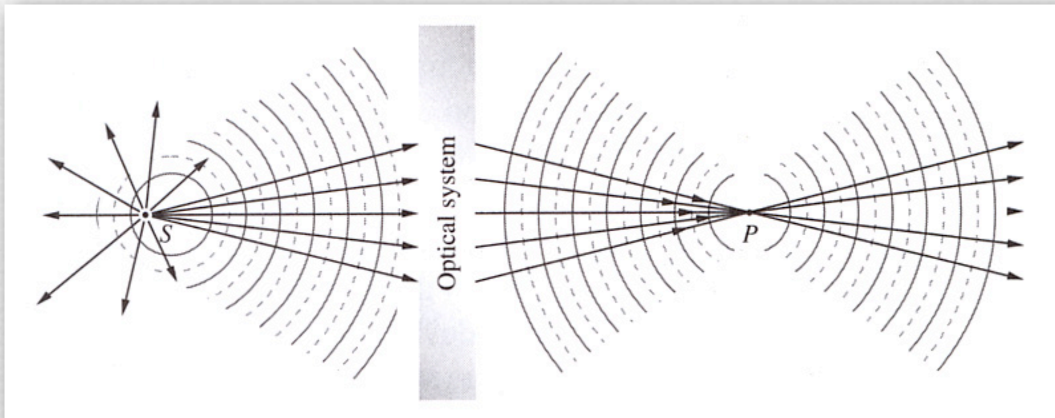
Replacing the pinhole with a lens





(London)

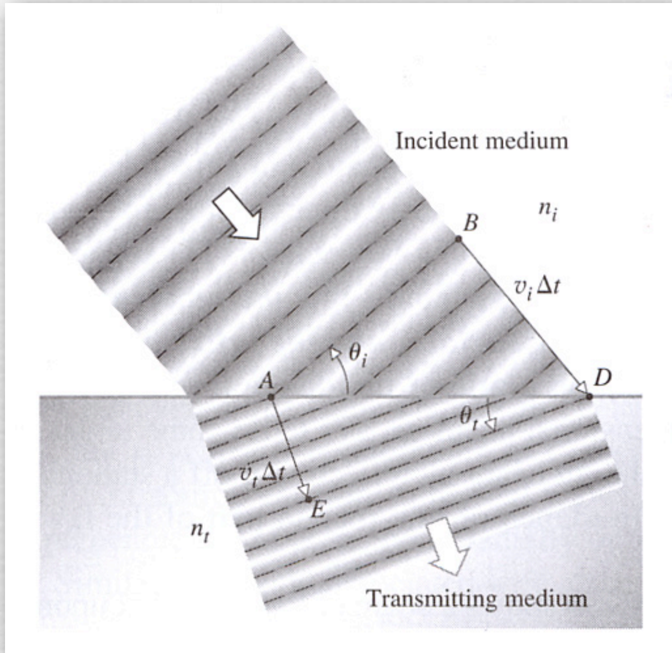
Physical versus geometrical optics



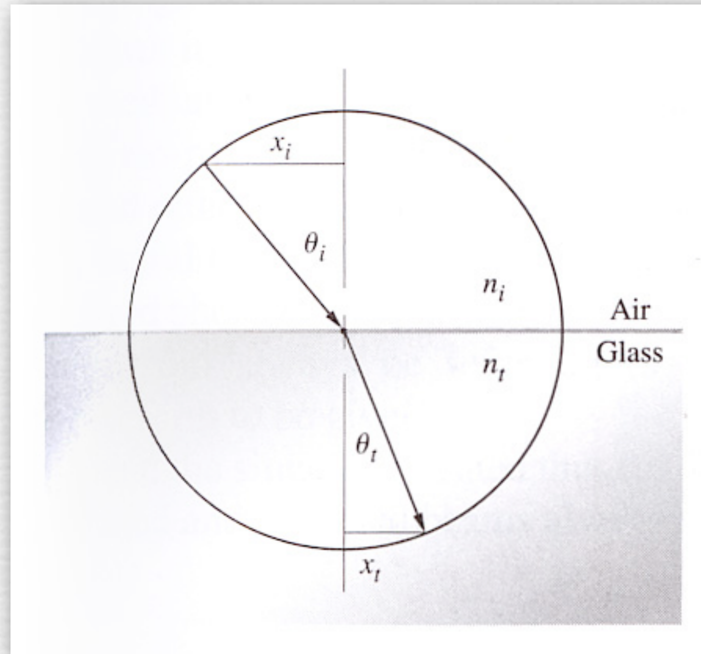
(Hecht)

- light can be modeled as traveling waves
- the perpendiculars to these waves can be drawn as rays
- diffraction causes these rays to bend, e.g. at a slit
- *geometrical optics* assumes
  - $\lambda \to 0$
  - no diffraction
  - in free space, rays are straight (a.k.a. rectilinear propagation)

Snell's law of refraction



(Hecht)



$$\frac{x_i}{x_t} = \frac{\sin\theta_i}{\sin\theta_t} = \frac{n_t}{n_i}$$

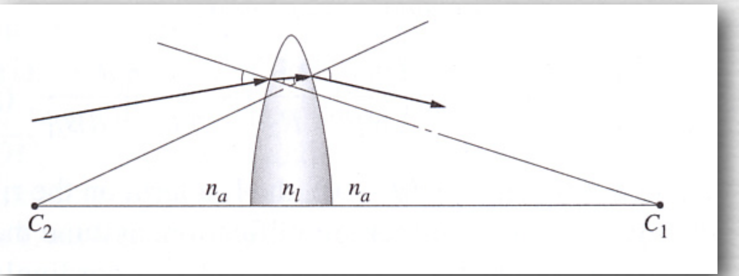
$$n \sin i = n' \sin i'$$

- as waves change speed at an interface, they also change direction
- index of refraction *n* is defined as the ratio between the speed of light in a vaccum / speed in some medium

Typical refractive indices (n)

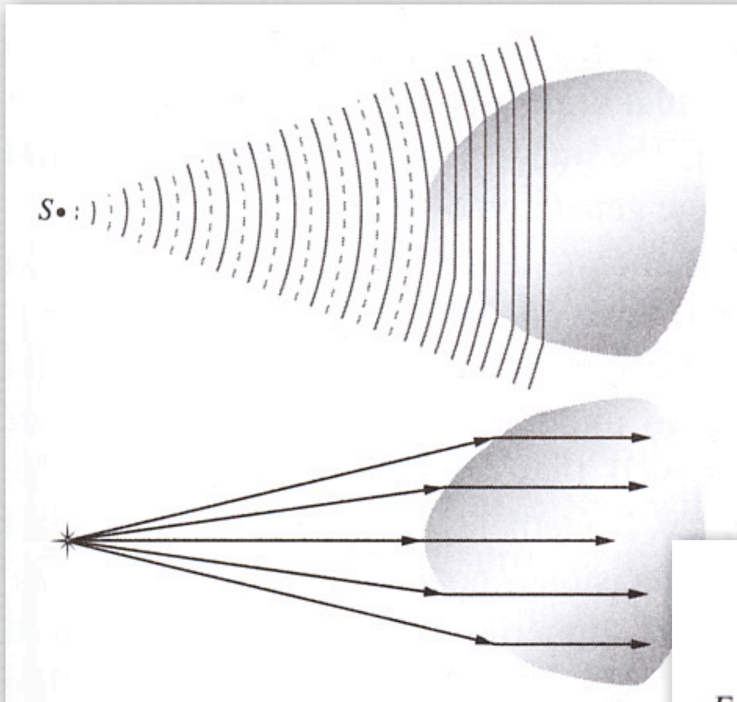
- air = 1.0
- water = 1.33
- glass = 1.5 - 1.8

- when transiting from air to glass, light bends towards the normal
- when transiting from glass to air, light bends away from the normal
- light striking a surface perpendicularly does not bend

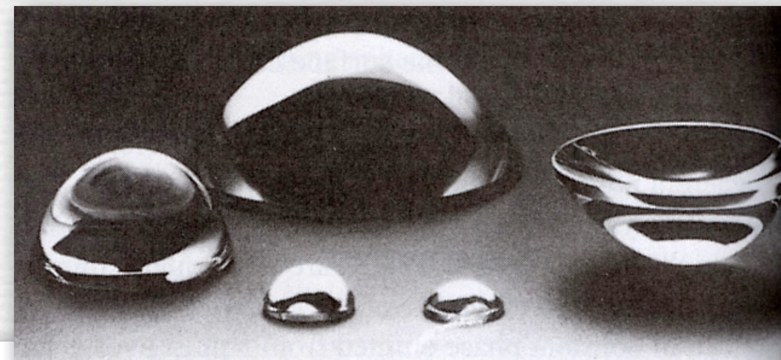


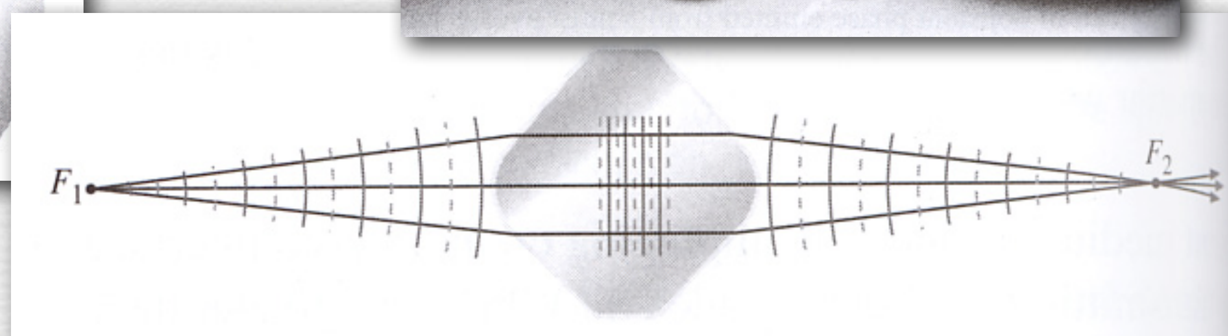
(Hecht)

Q. What shape should an interface be to make parallel rays converge to a point?



(Hecht)



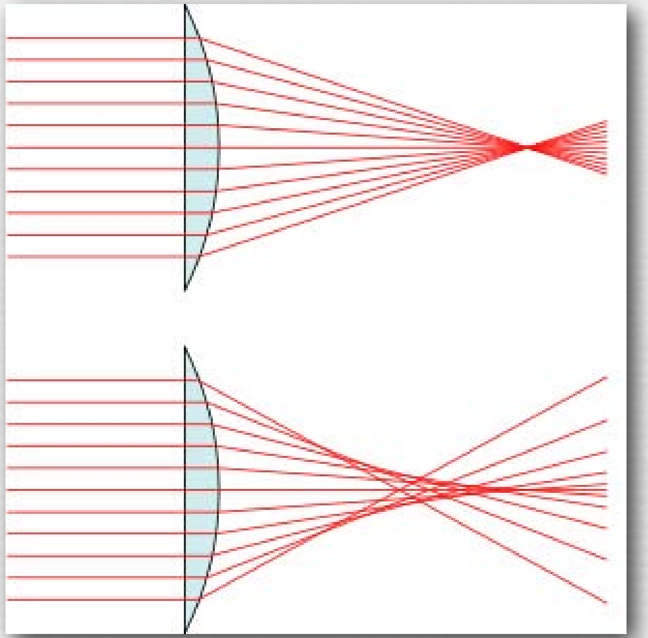


A. a hyperbola

- so lenses should be hyperbolic!

Spherical lenses

(Hecht)



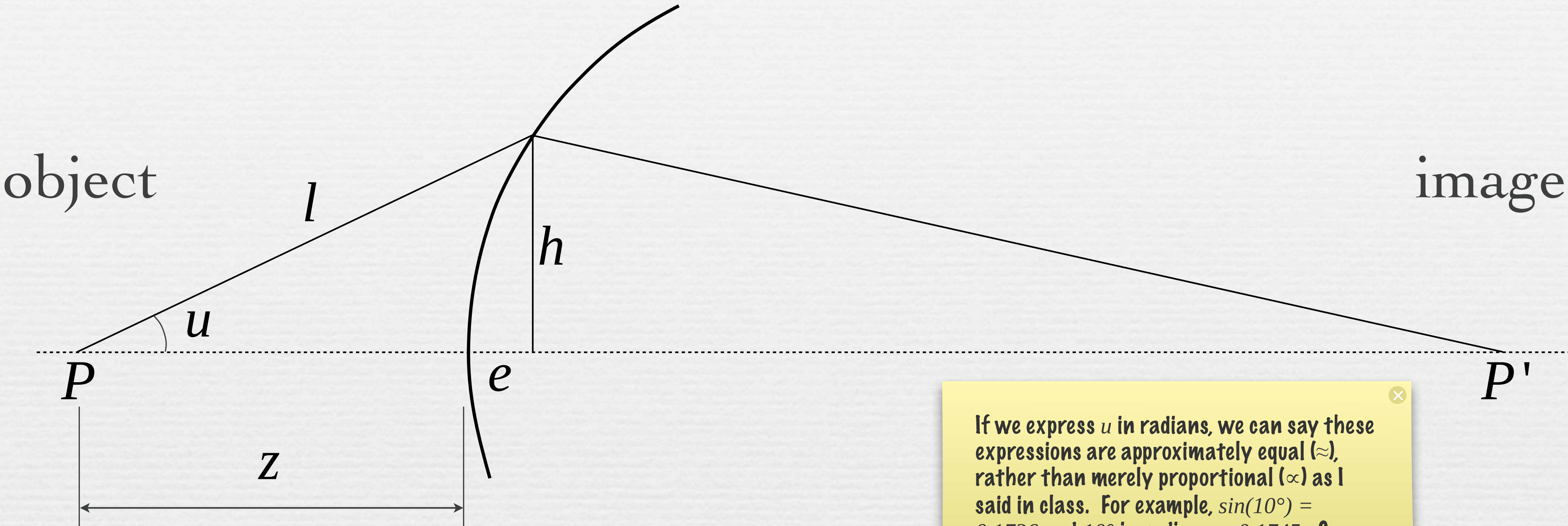
(wikipedia)

- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don't bring parallel rays to a point
  - this is called *spherical aberration*
  - nearly axial rays *(paraxial rays)* behave best

Paraxial approximation

- assume $e \approx 0$

Paraxial approximation

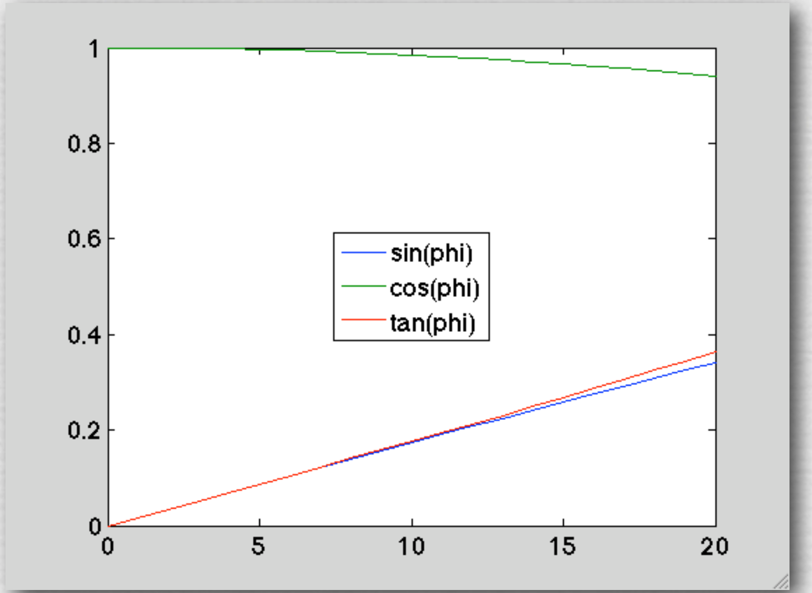


If we express u in radians, we can say these expressions are approximately equal (≈), rather than merely proportional (∝) as I said in class. For example, $\sin(10°) = 0.1736$ and $10°$ in radians $= 0.1745$. See how close these values are? In keeping with this cleaner explanation, I've changed all "=" and "∝" to "≈" in this sequence of slides.

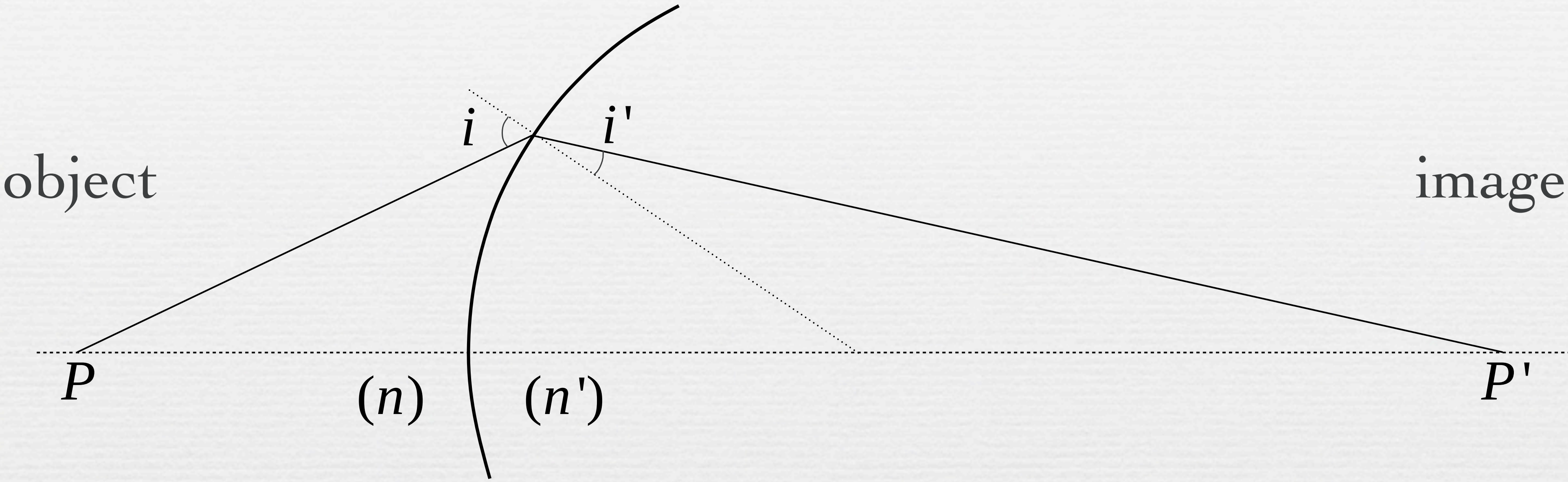
- assume $e \approx 0$
- assume $\sin u = h / l \approx u$ (for u in radians)
- assume $\cos u \approx z / l \approx 1$
- assume $\tan u \approx \sin u \approx u$

The paraxial approximation is a.k.a. first-order optics

- assume first term of $\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots$
  - i.e. $\sin\phi \approx \phi$
- assume first term of $\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots$
  - i.e. $\cos\phi \approx 1$
  - so $\tan\phi \approx \sin\phi \approx \phi$



Paraxial focusing



Snell's law:

$$n \sin i = n' \sin i'$$

paraxial approximation:

$$n\, i \approx n'\, i'$$

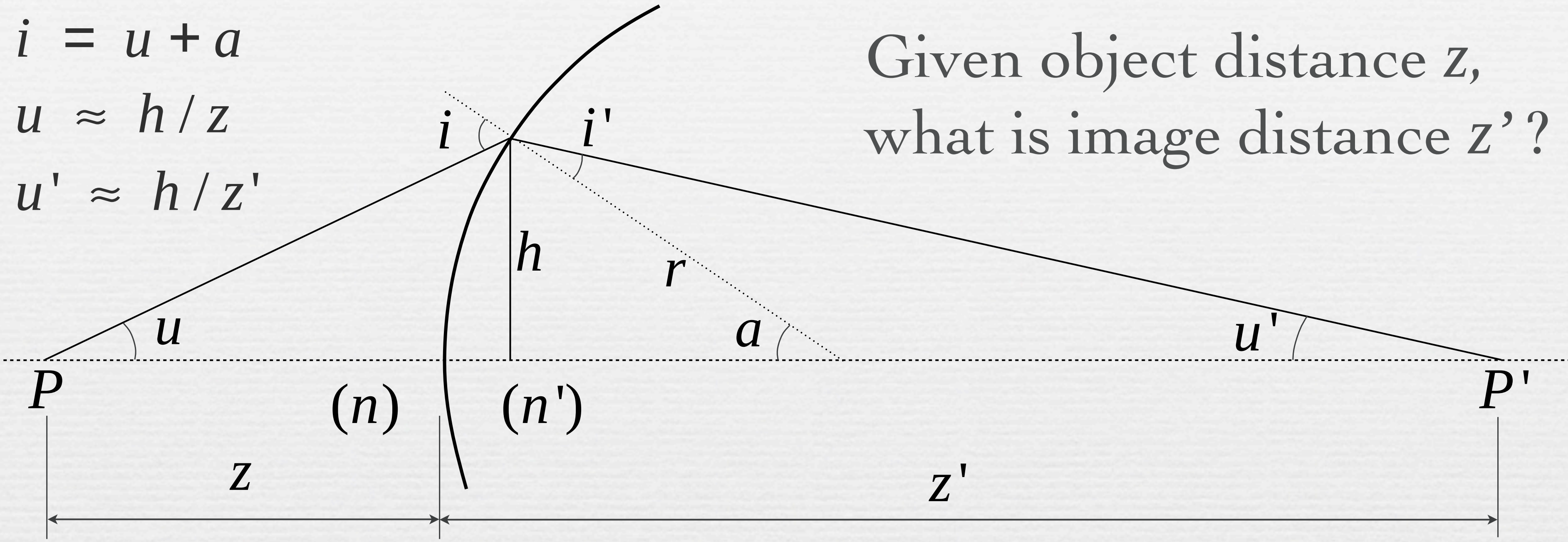
Paraxial focusing

$i = u + a$

$u \approx h / z$

$u' \approx h / z'$

Given object distance z, what is image distance z'?



$n\, i \approx n'\, i'$

Paraxial focusing

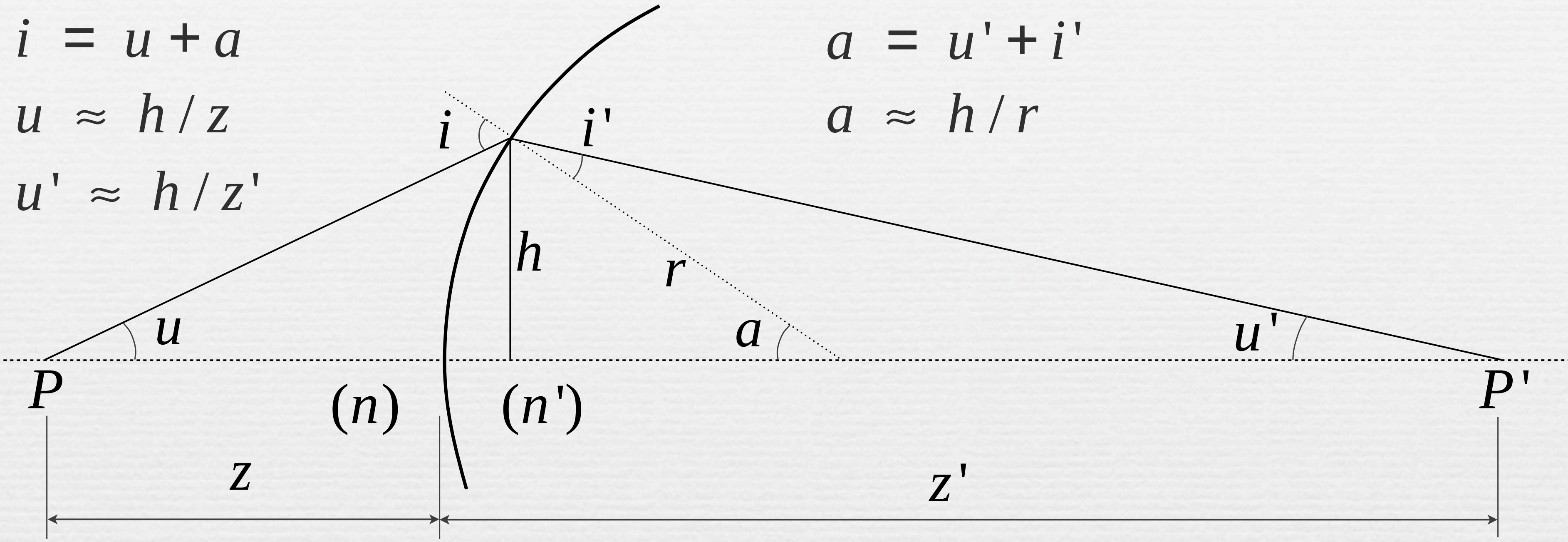
$i = u + a$

$u \approx h/z$

$u' \approx h/z'$

$a = u' + i'$

$a \approx h/r$



$n(u + a) \approx n'(a - u')$

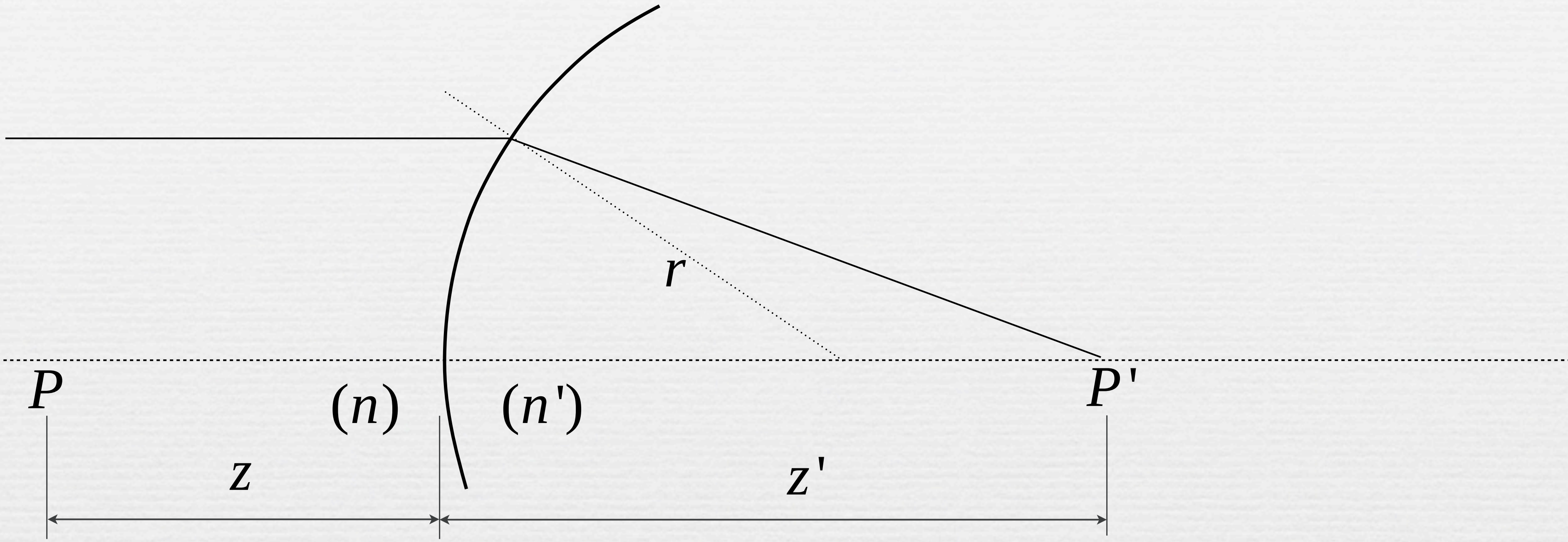
$n(h/z + h/r) \approx n'(h/r - h/z')$

$n/z + n/r \approx n'/r - n'/z'$

$n\,i \approx n'\,i'$

- *h* has canceled out, so any ray from *P* will focus to *P'*

Focal length



What happens if z is ∞ ?

$$n / z + n / r \approx n' / r - n' / z'$$

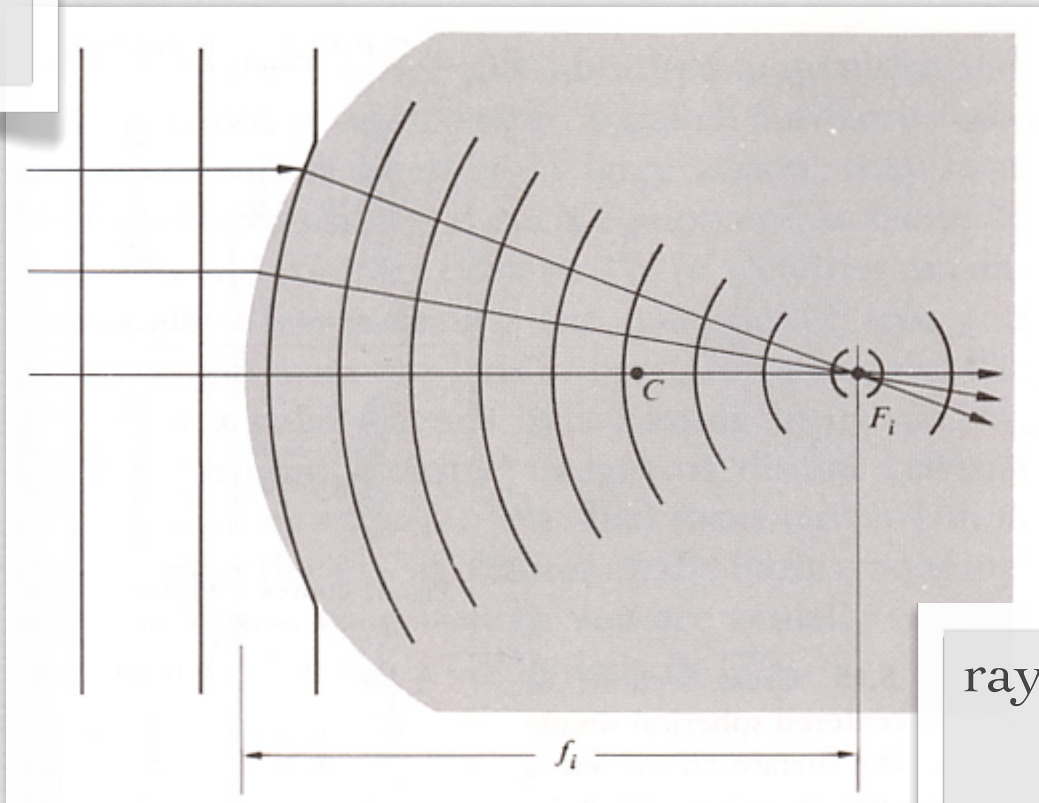
$$n / r \approx n' / r - n' / z'$$

$$z' \approx (r\, n') / (n' - n)$$

- $f \triangleq$ focal length = z'

Focusing of rays versus waves

rays from infinity ≡ plane waves

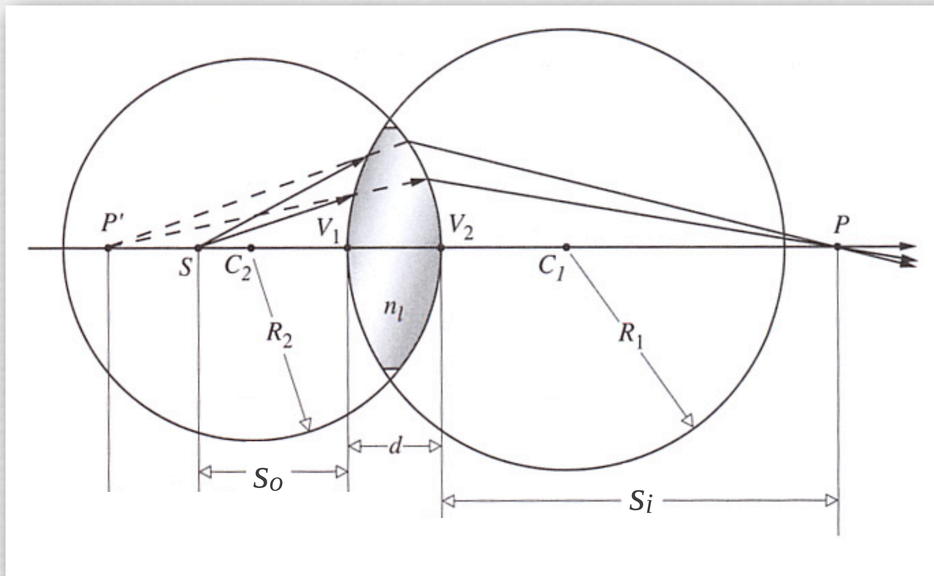


(Hecht)

rays converging to a focus ≡ spherical waves

Lensmaker's formula

- using similar derivations, one can extend these results to two spherical interfaces forming a lens in air



(Hecht, edited)

- as $d \to 0$ *(thin lens approximation)*, we obtain the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Gaussian lens formula

- Starting from the lensmaker's formula

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad \text{(Hecht, eqn 5.15)}$$

- and recalling that as object distance s_o is moved to infinity, image distance s_i becomes focal length f_i , we get

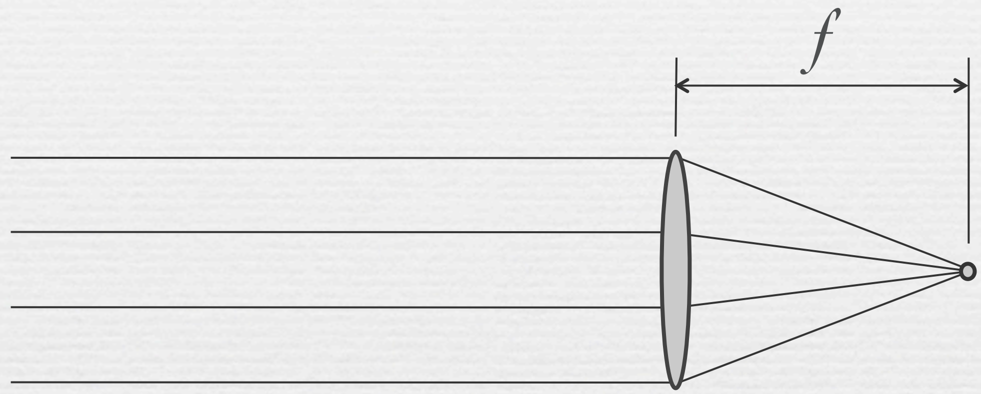
$$\frac{1}{f_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \qquad \text{(Hecht, eqn 5.16)}$$

- Equating these two, we get the Gaussian lens formula

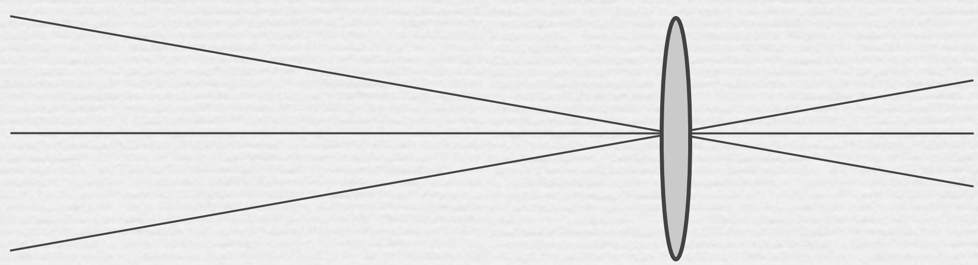
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_i}. \qquad \text{(Hecht, eqn 5.17)}$$

Gauss' ray tracing construction

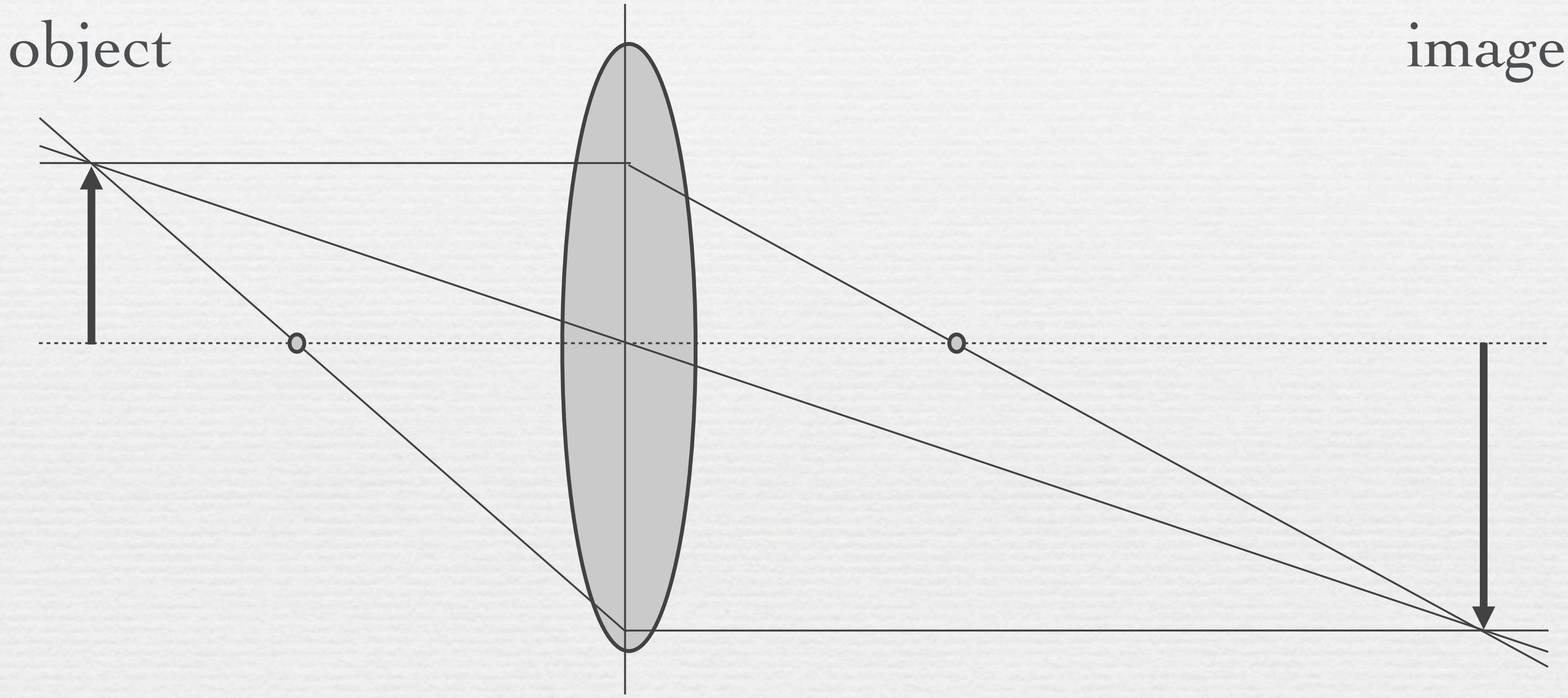
- assume that parallel rays converge to a point located at focal length f from lens



- and rays going through center of lens are not deviated
  - hence same perspective as pinhole

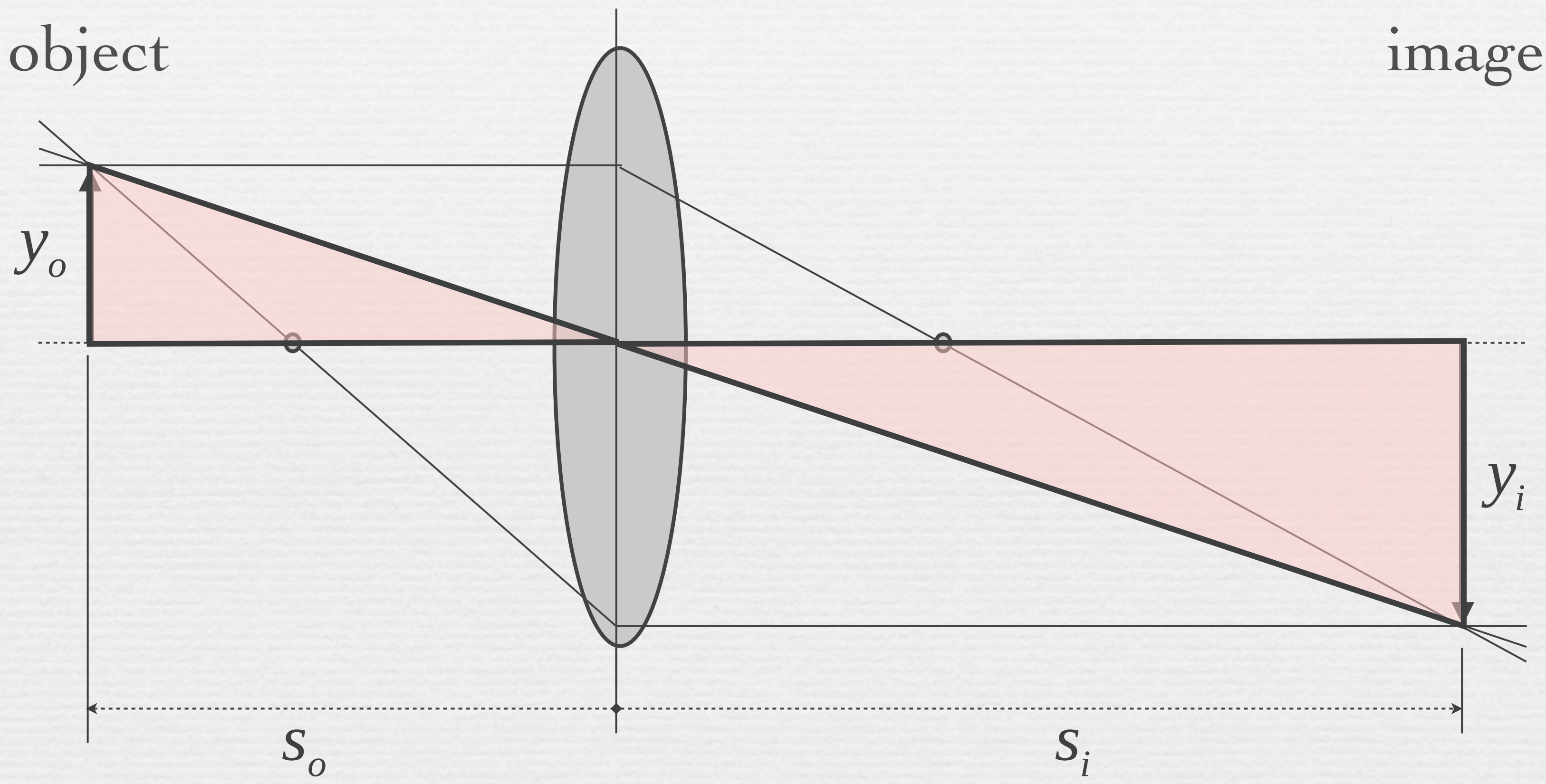


Gauss' ray tracing construction



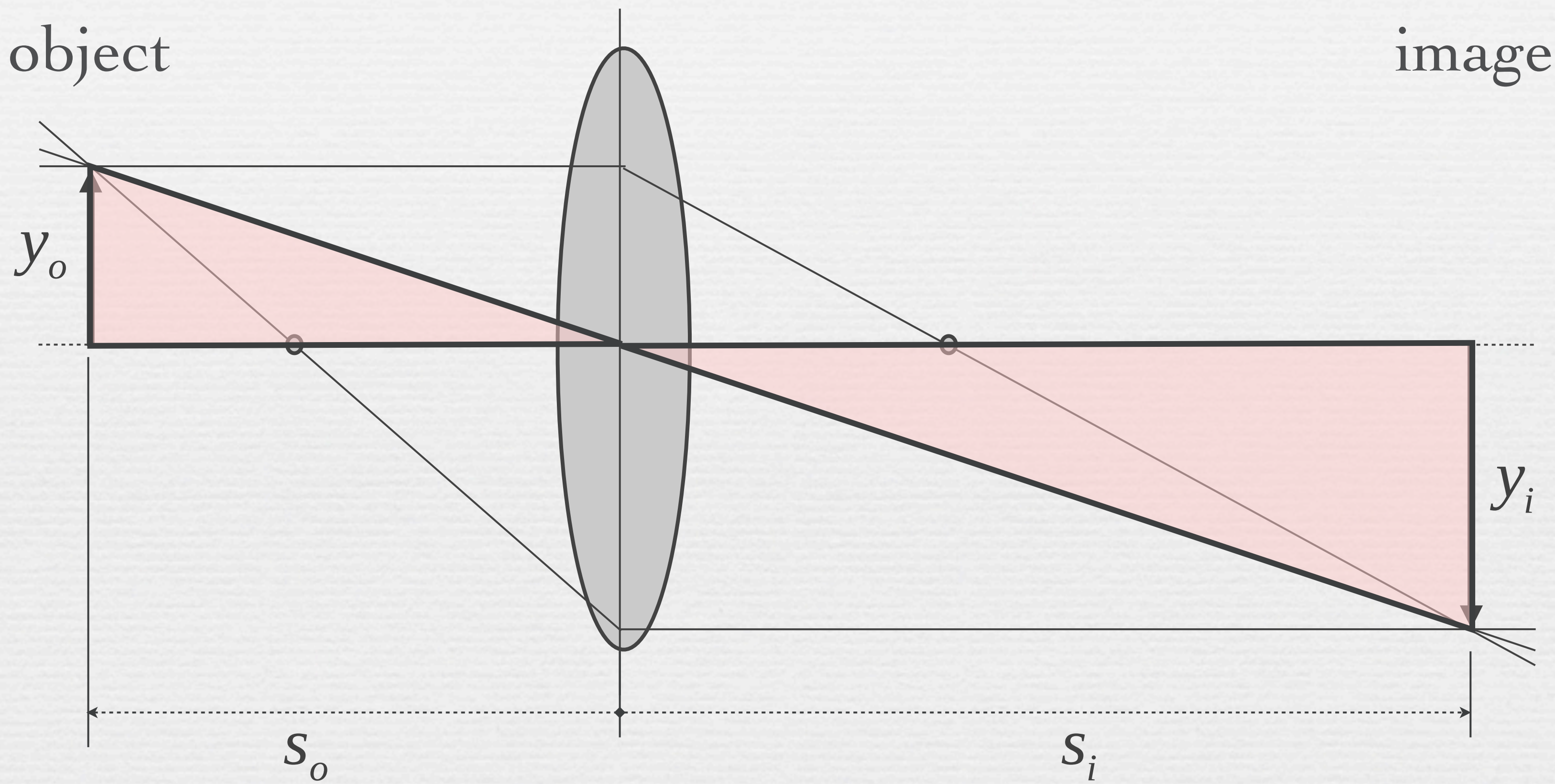
- rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens

From Gauss's ray construction to the Gaussian lens formula



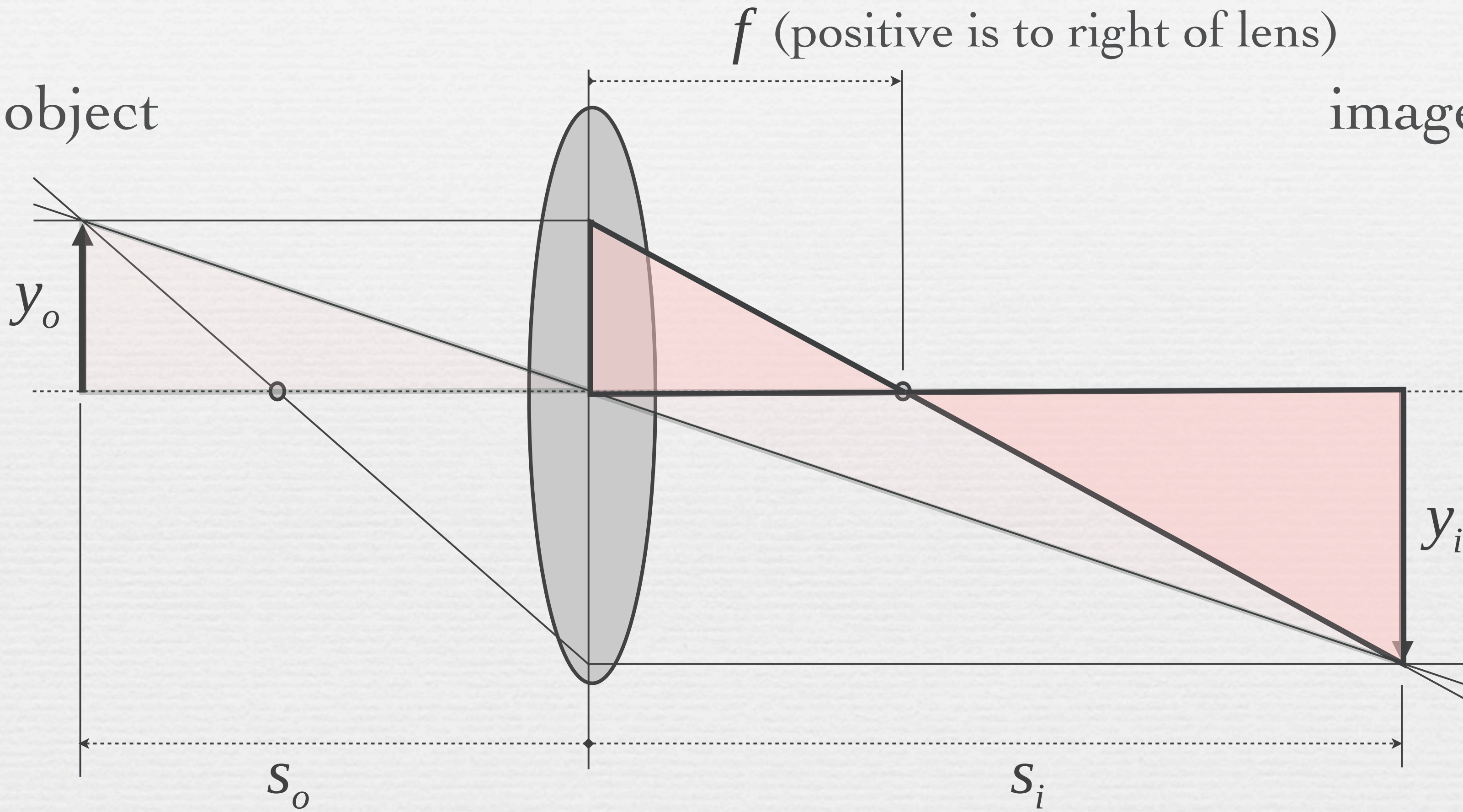
- positive s_i is rightward, positive s_o is leftward
- positive y is upward

From Gauss's ray construction to the Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$

From Gauss's ray construction to the Gaussian lens formula

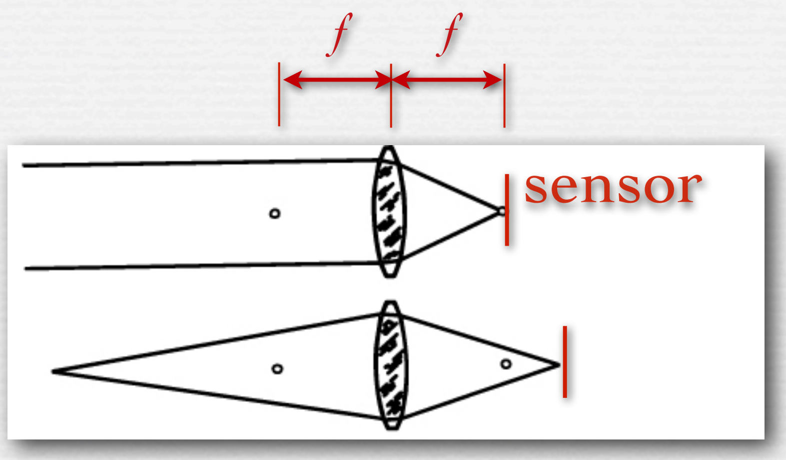


$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \ldots\ldots$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

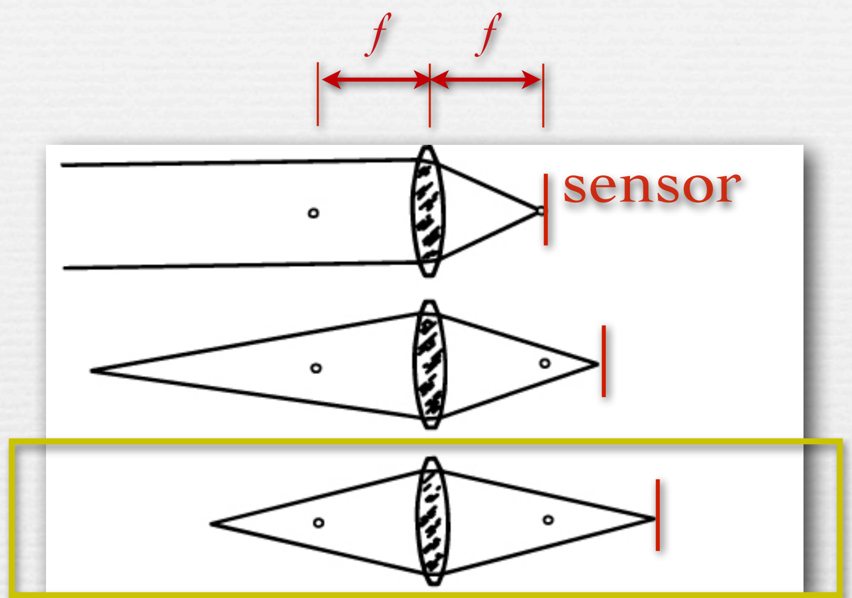
- to focus on objects
  at different distances,
  move sensor relative to lens



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens



- at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

In 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame.

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

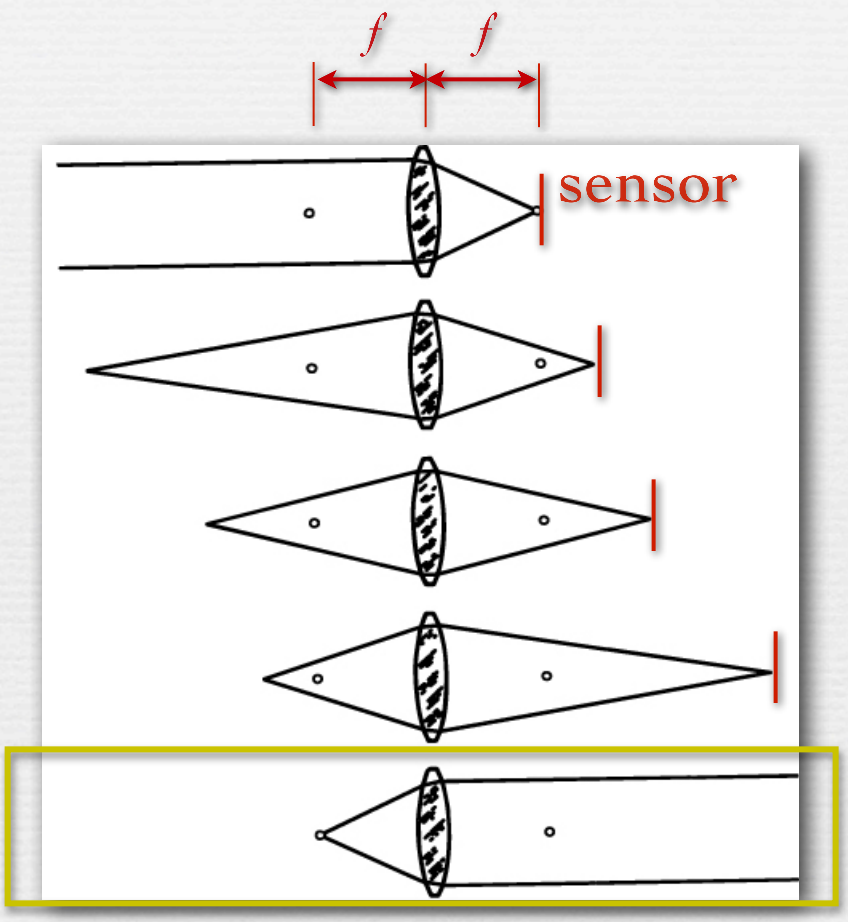
Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens

- at $s_o = s_i = 2f$ we have 1:1 imaging, because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

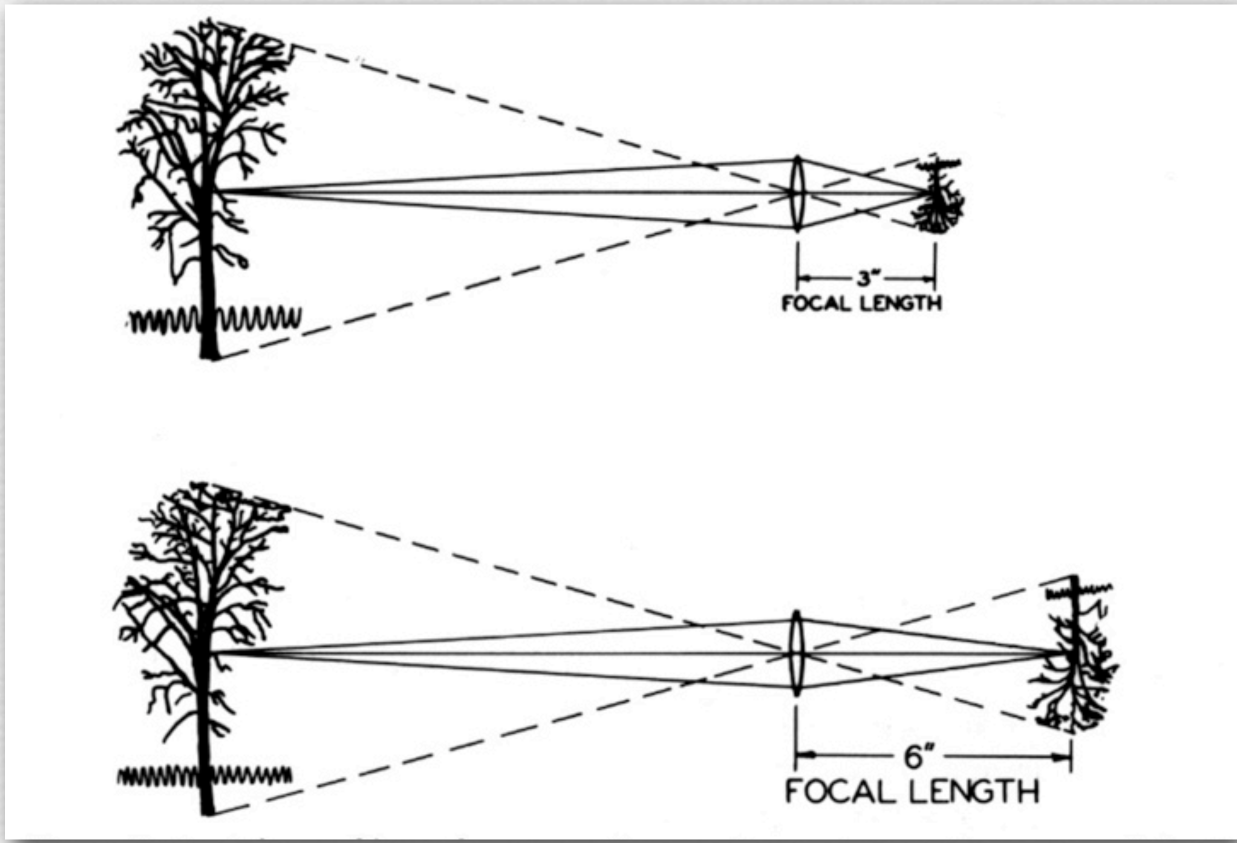
- can't focus on objects closer to lens than its focal length f



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Changing the focal length

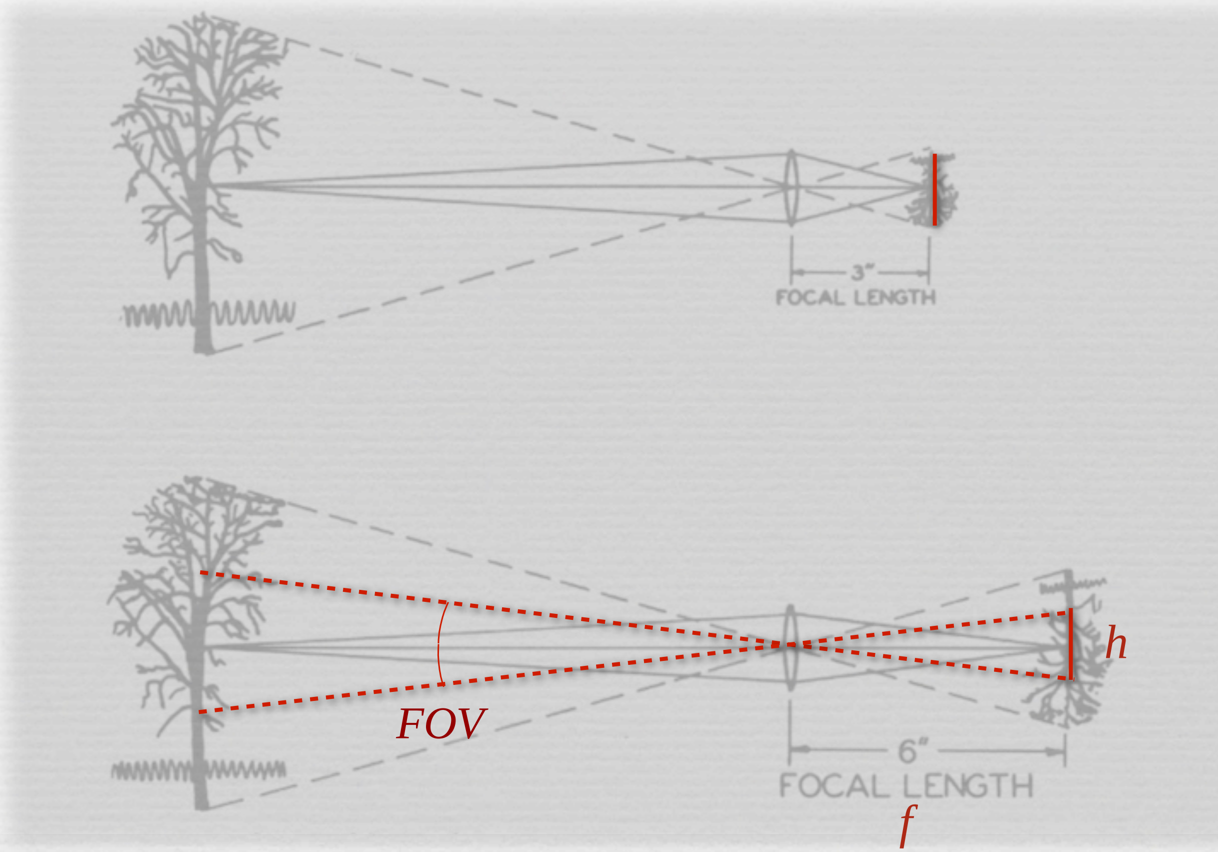
- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back



(Kingslake)

Changing the focal length

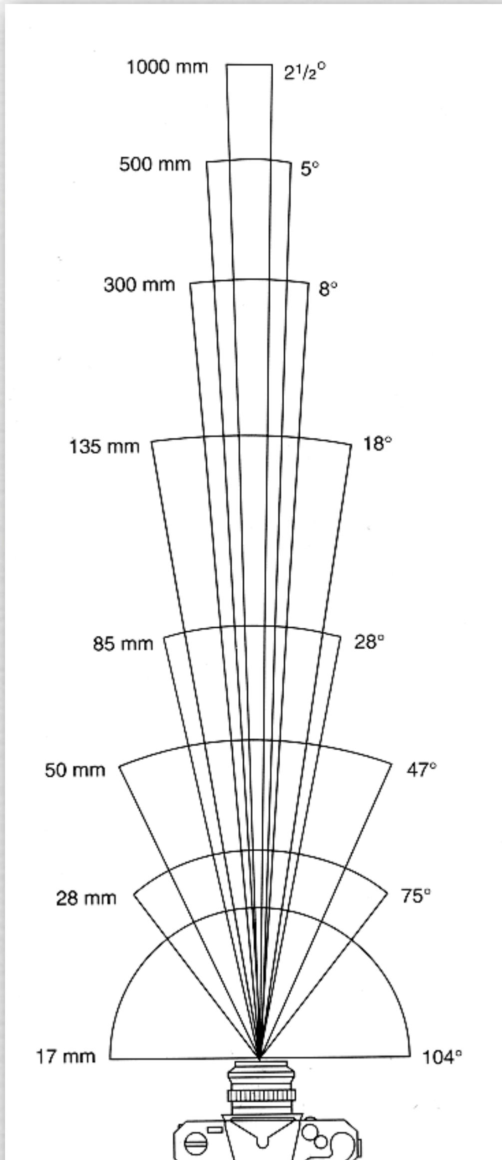
- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back
- if the sensor size is constant, the field of view becomes smaller

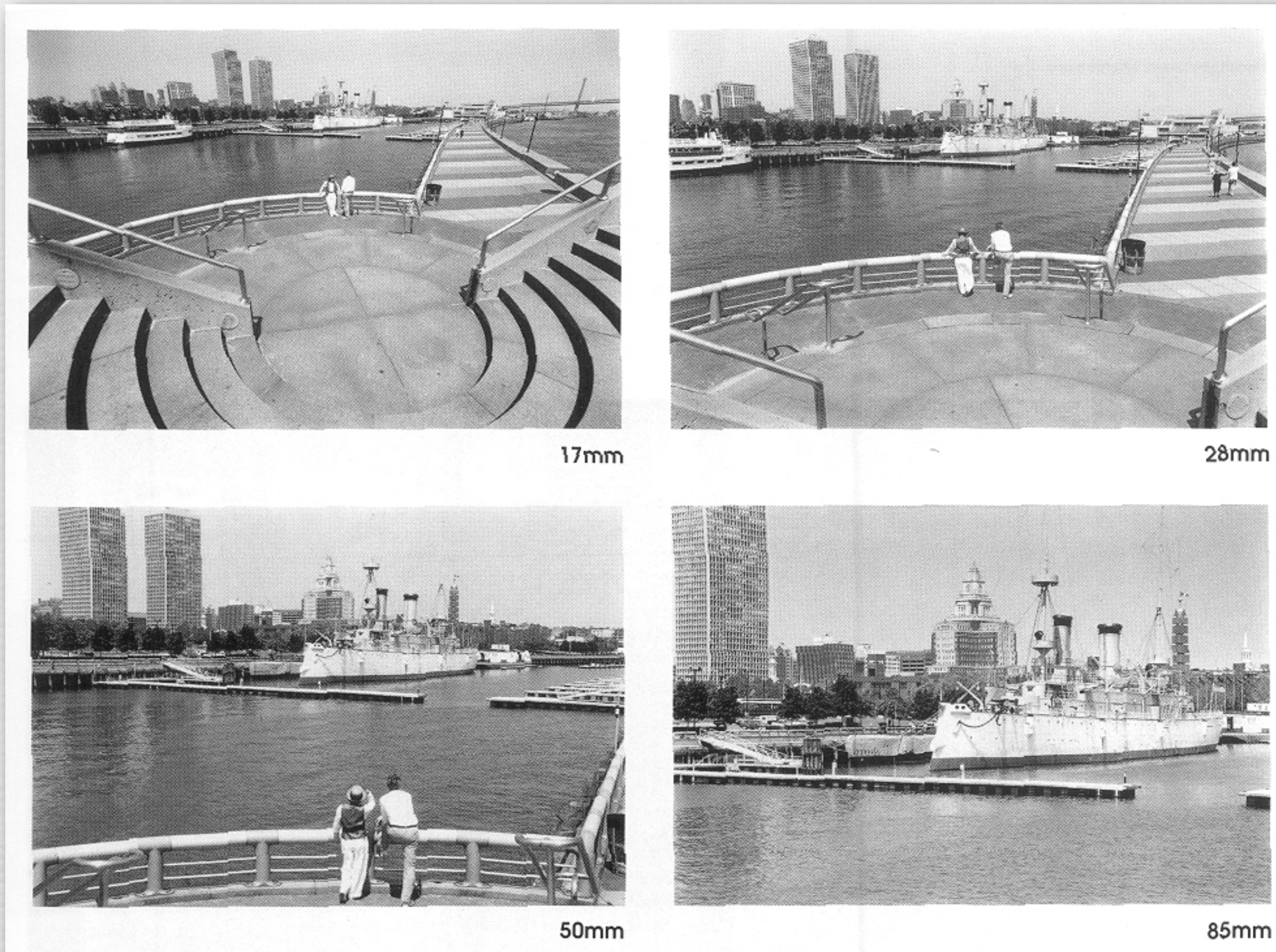


(Kingslake)

$$FOV = 2 \arctan(h / 2f)$$

Focal length and field of view

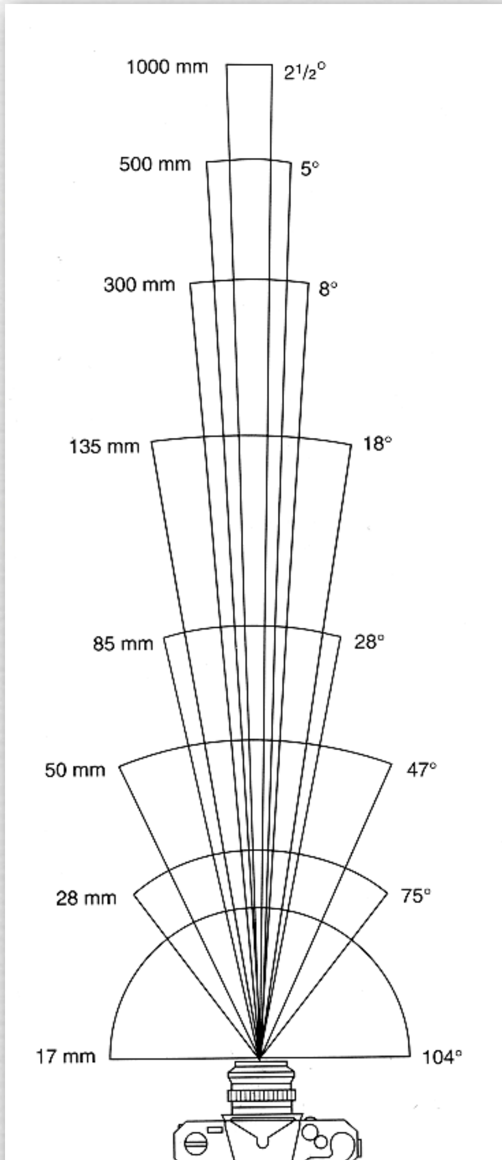


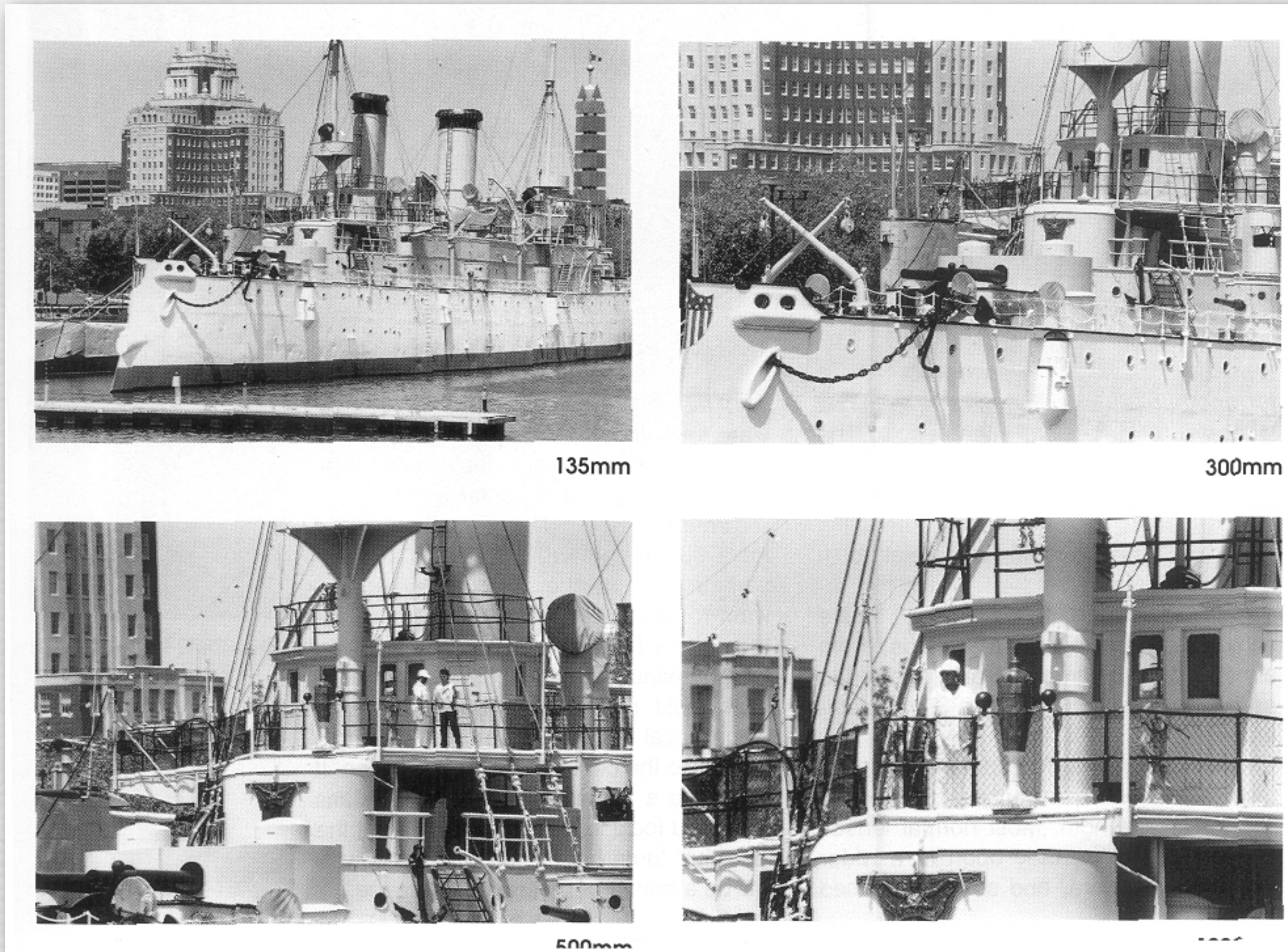


(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

Focal length and field of view



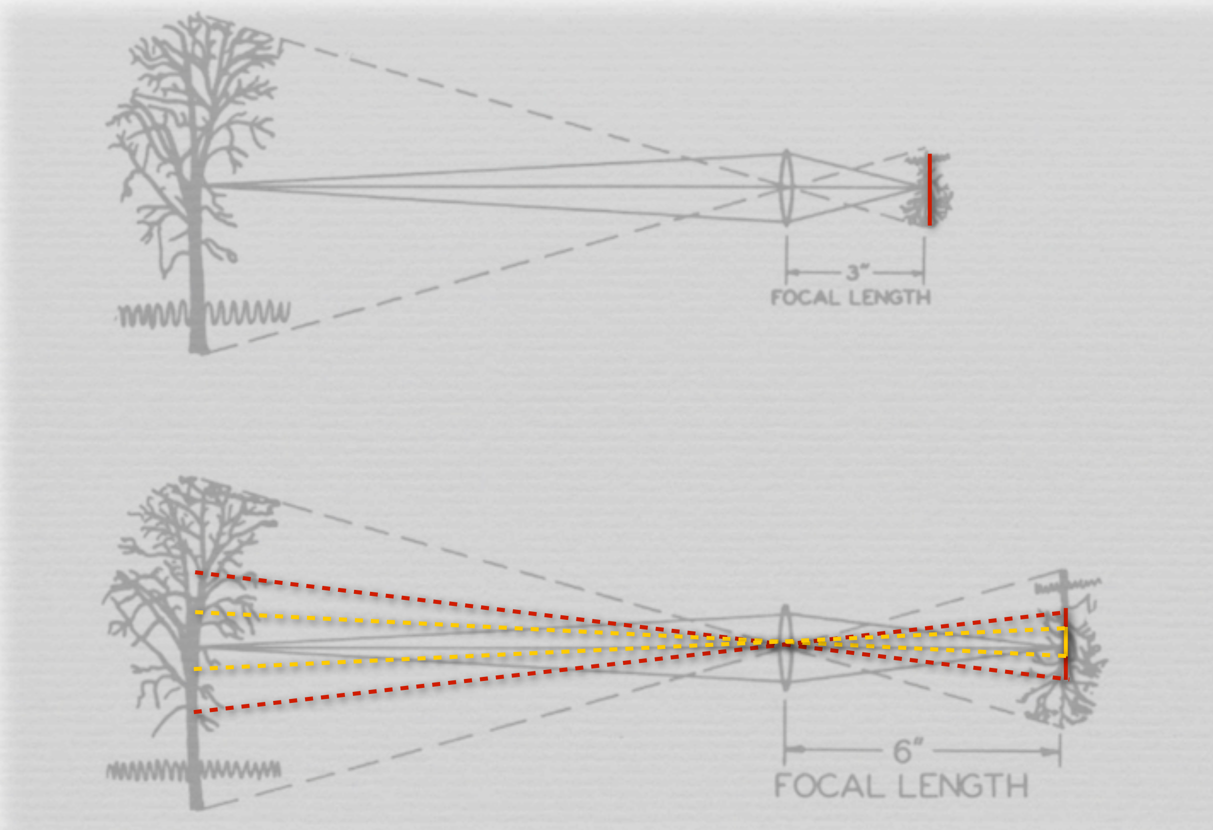


(London)

FOV measured diagonally on a
35mm full-frame camera (24 × 36mm)

Changing the sensor size

- if the sensor size is smaller, the field of view is smaller too
- smaller sensors either have fewer pixels, or noiser pixels

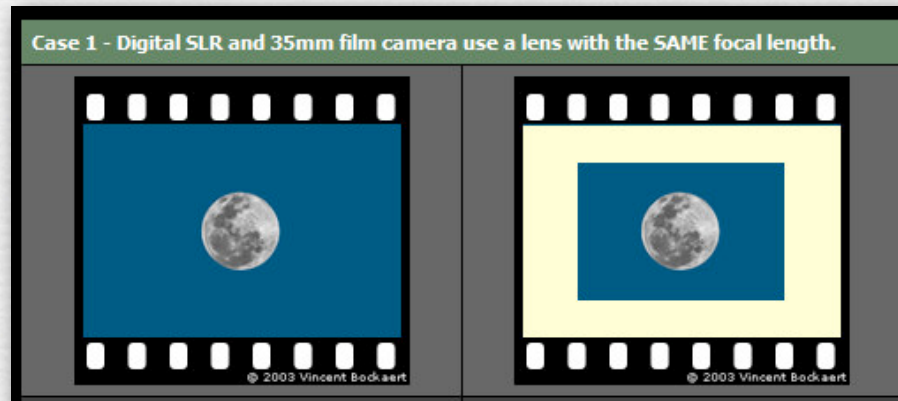


(Kingslake)

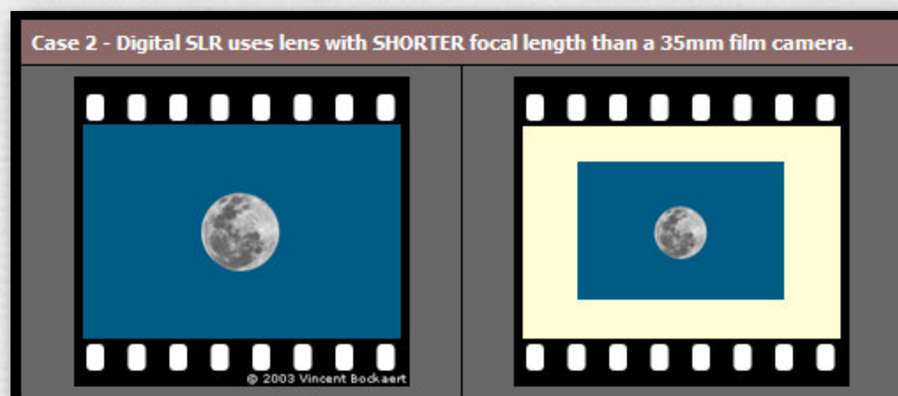
Full-frame 35mm versus APS-C

- full-frame sensor is 24 × 36mm (same as 35mm film)
- APS-C sensor is 14.8 × 22.2mm (Canon DSLRs)
- conversion factor is 1.6×

(dpreview)

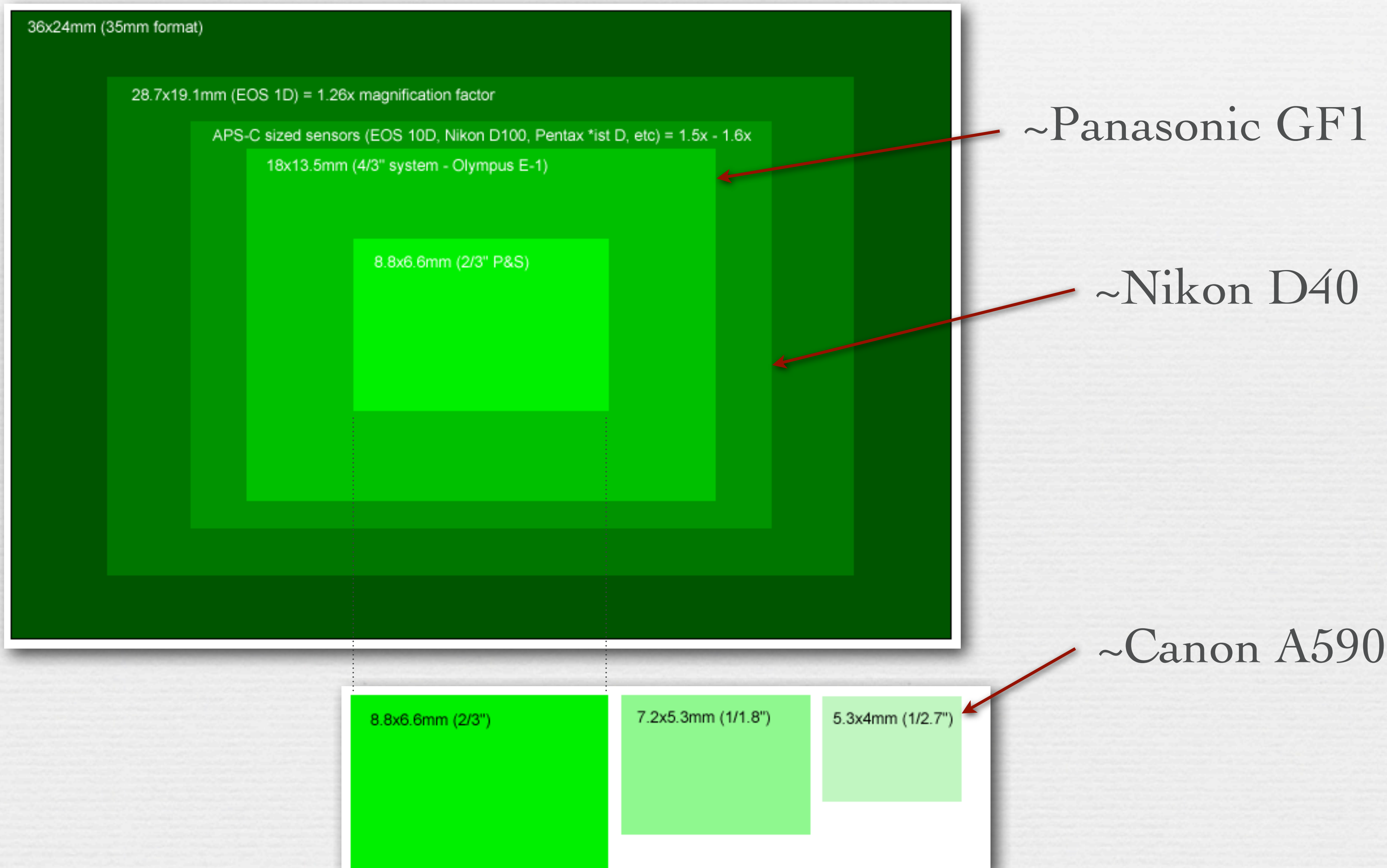


- switching camera bodies
  - object occupies the same number of pixels, but takes up more of frame



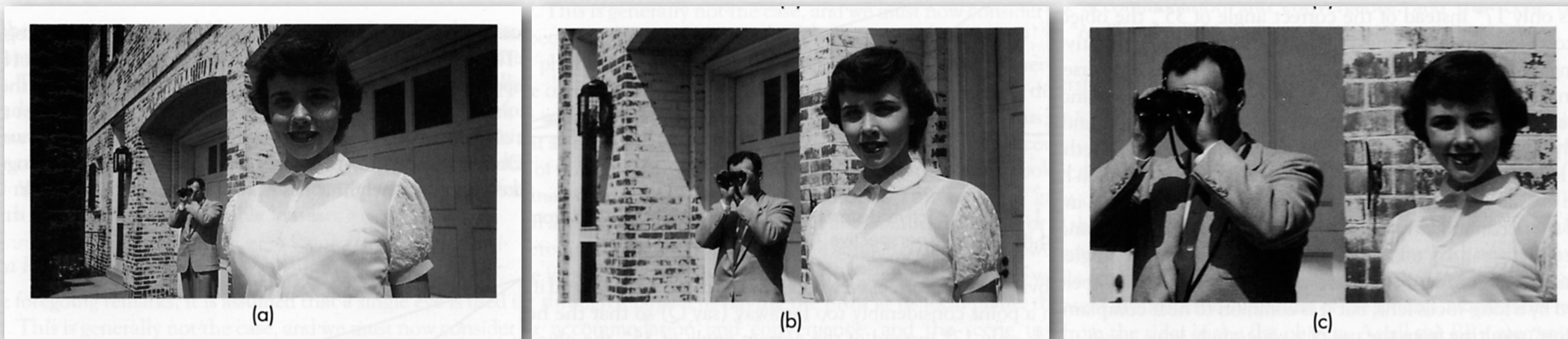
- switching lenses
  - objects occupies fewer pixels, but composition stays the same

Sensor sizes



Changing the focal length versus changing the viewpoint

(Kingslake)

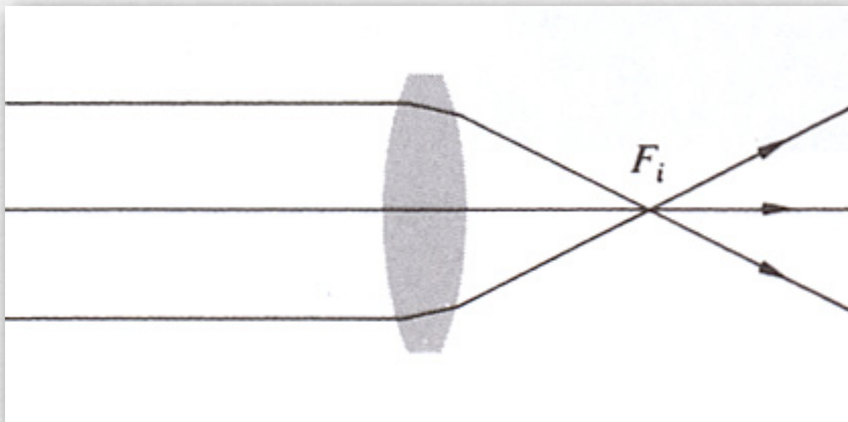


wide-angle telephoto

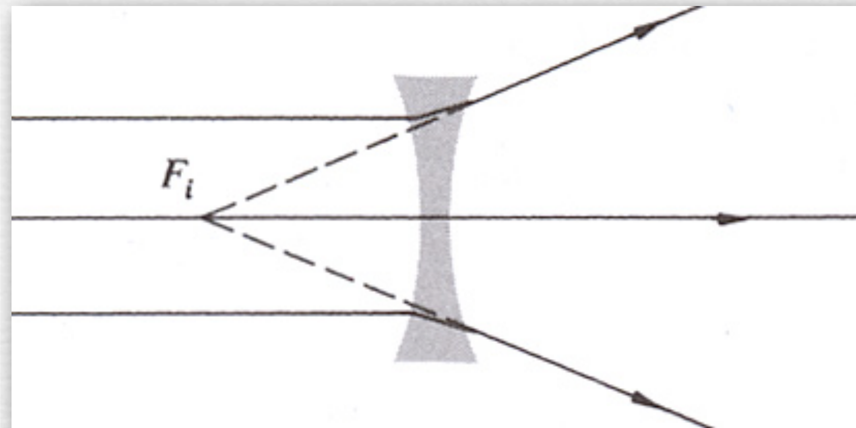
- changing the focal length lets us move back from a subject, while maintaining its size on the image
- but moving back changes perspective relationships

Convex versus concave lenses

(Hecht)



rays from a convex lens converge

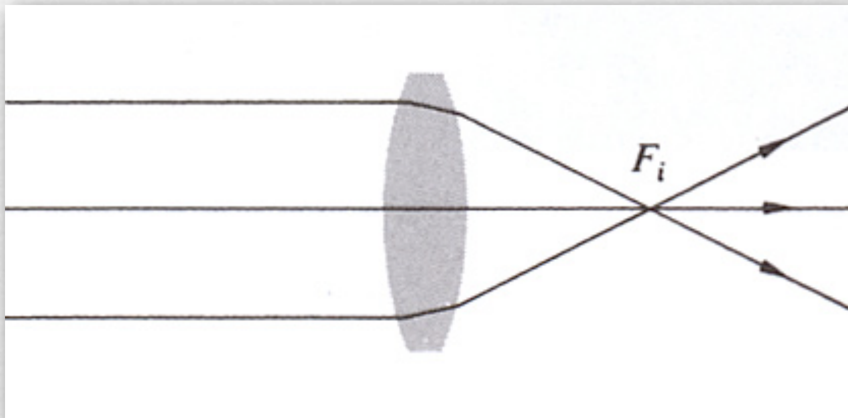


rays from a concave lens diverge

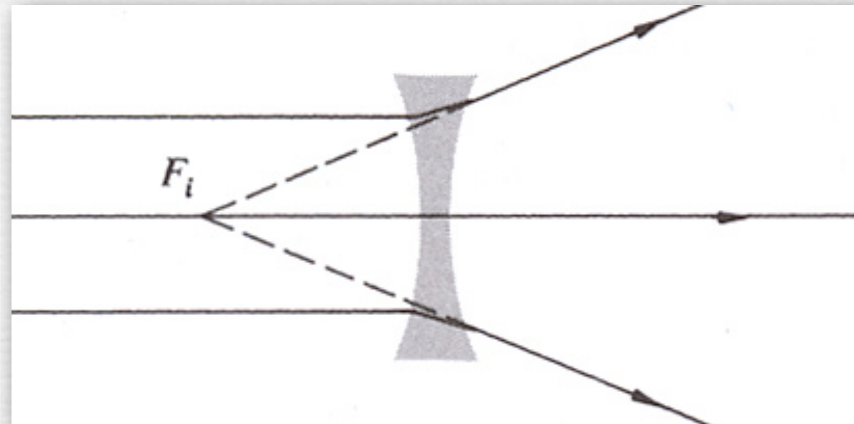
- positive focal length *f* means parallel rays from the left converge to a point on the right
- negative focal length *f* means parallel rays from the left converge to a point on the left (dashed lines above)

Convex versus concave lenses

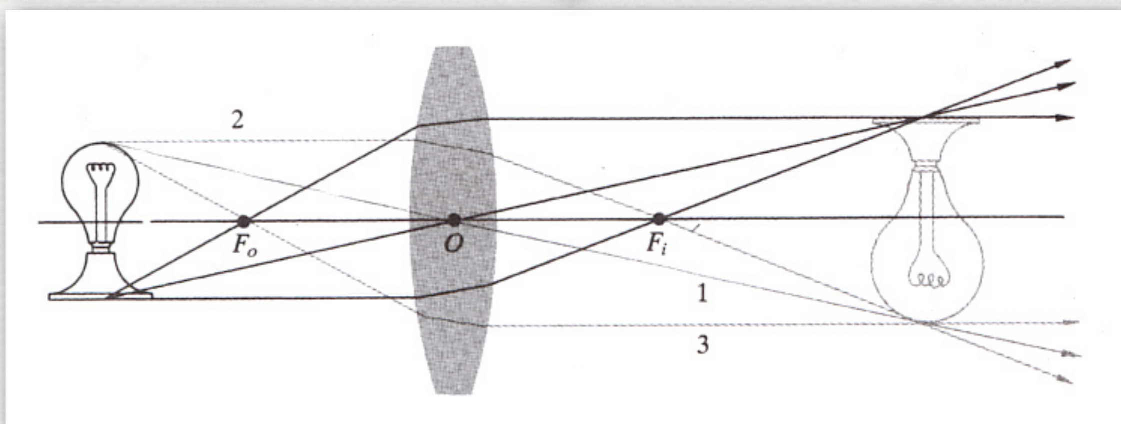
(Hecht)



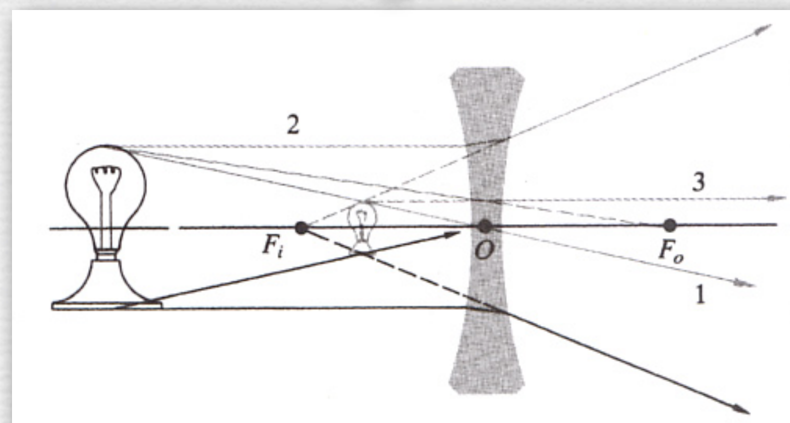
rays from a convex lens converge



rays from a concave lens diverge

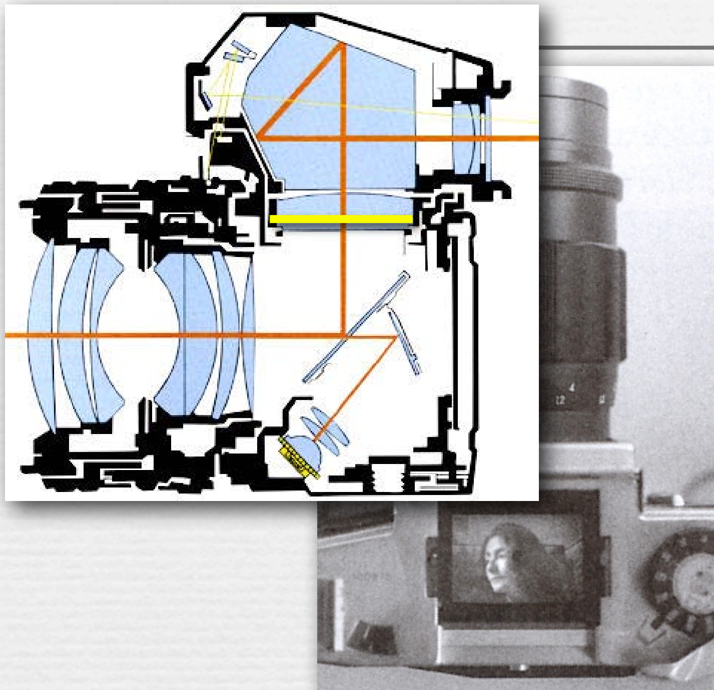


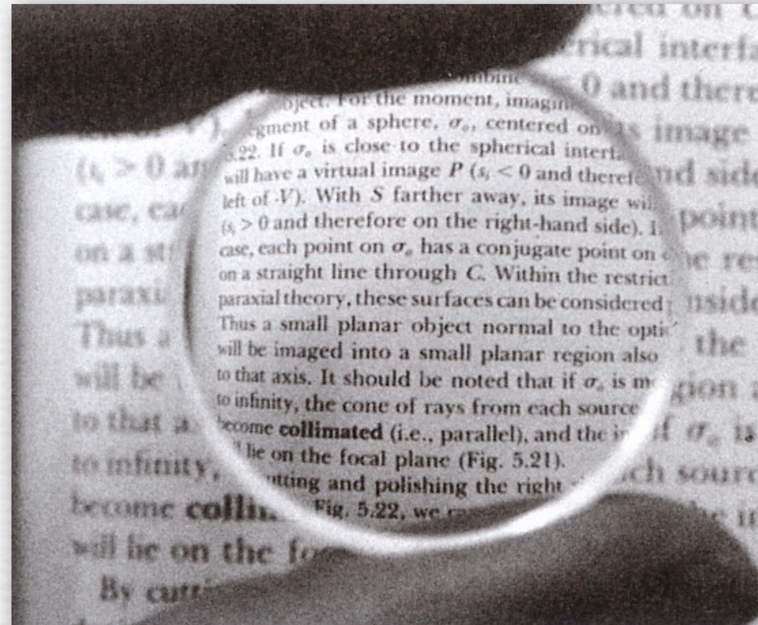
...producing a real image

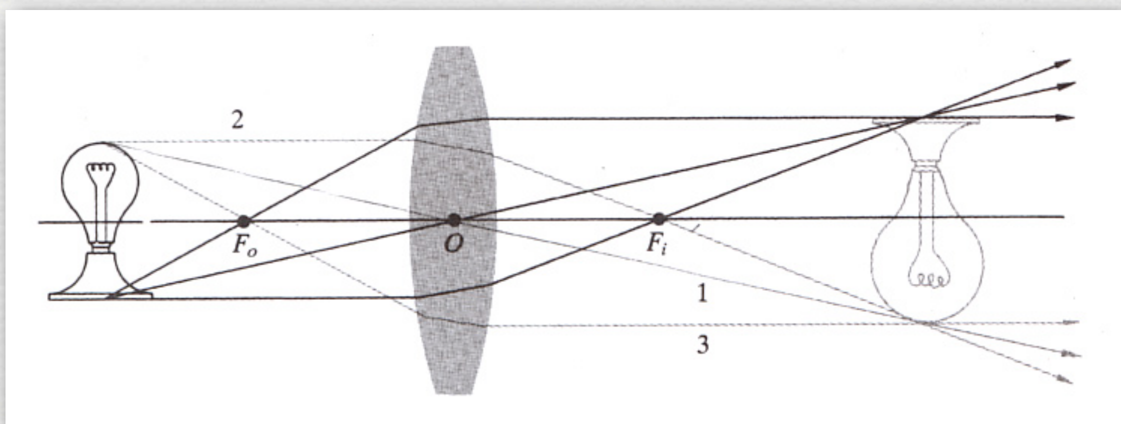


...producing a virtual image

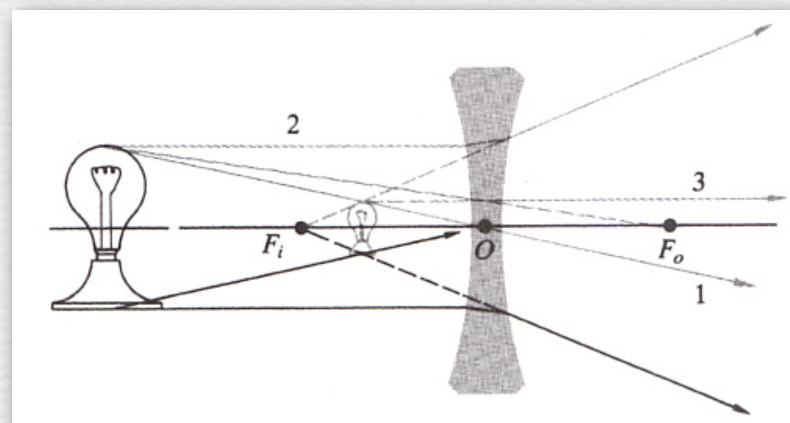
Convex versus concave lenses







...producing a real image



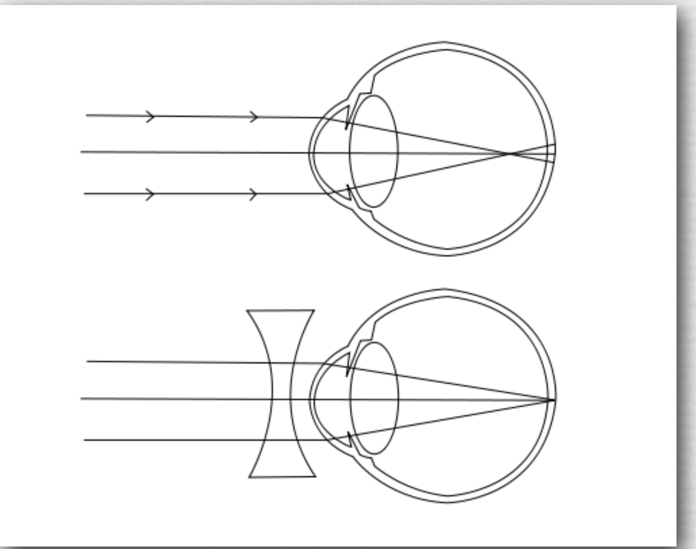
...producing a virtual image

The power of a lens

$$P = \frac{1}{f}$$

- units are meters^{-1}
- a.k.a. diopters

(wikipedia)

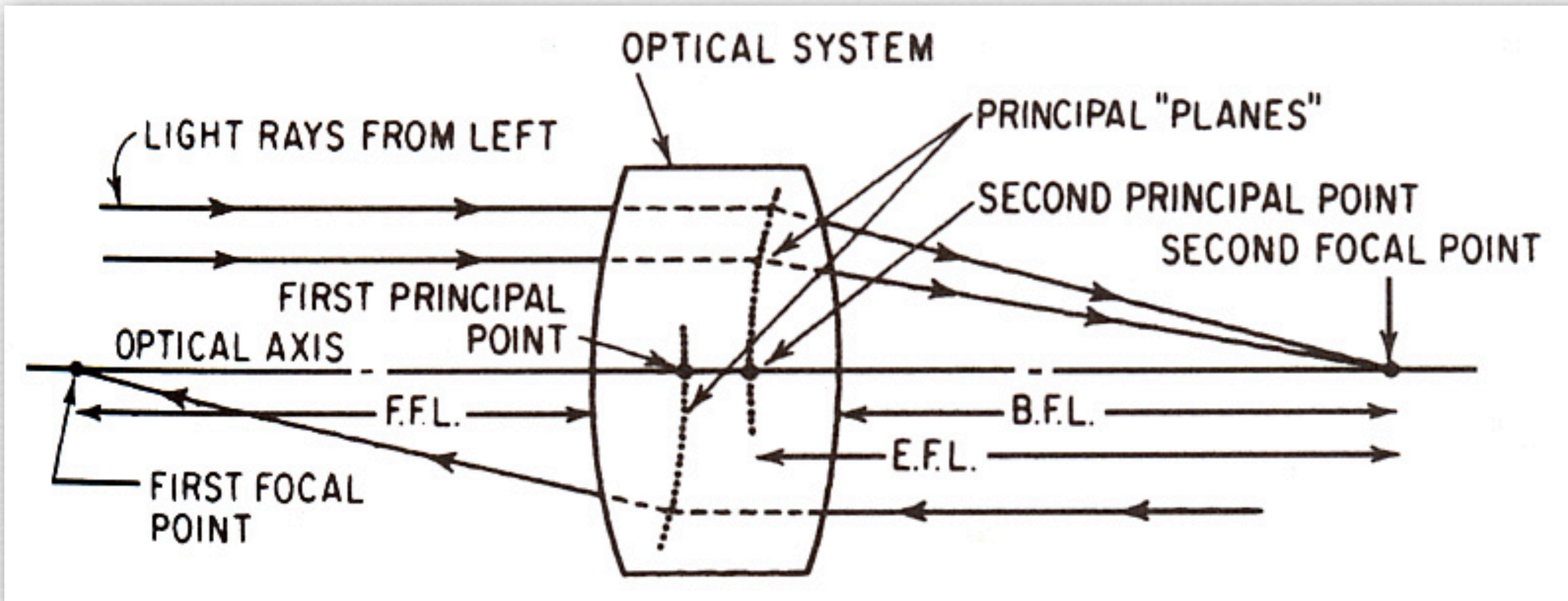


- my eyeglasses have the prescription
  - right eye: -0.75 diopters
  - left eye: -1.00 diopters

Q. What's wrong with me?
A. Myopia

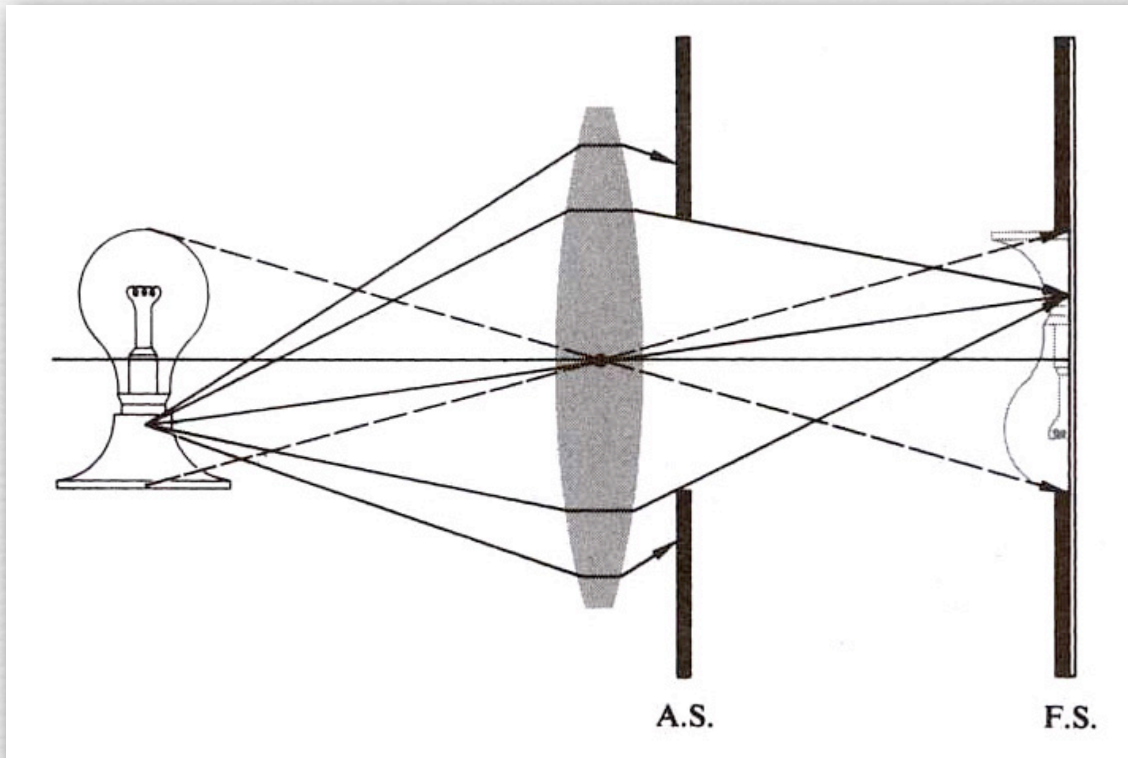
Thick lenses

- an optical system may contain many lenses, but can be characterized by a few numbers



(Smith)

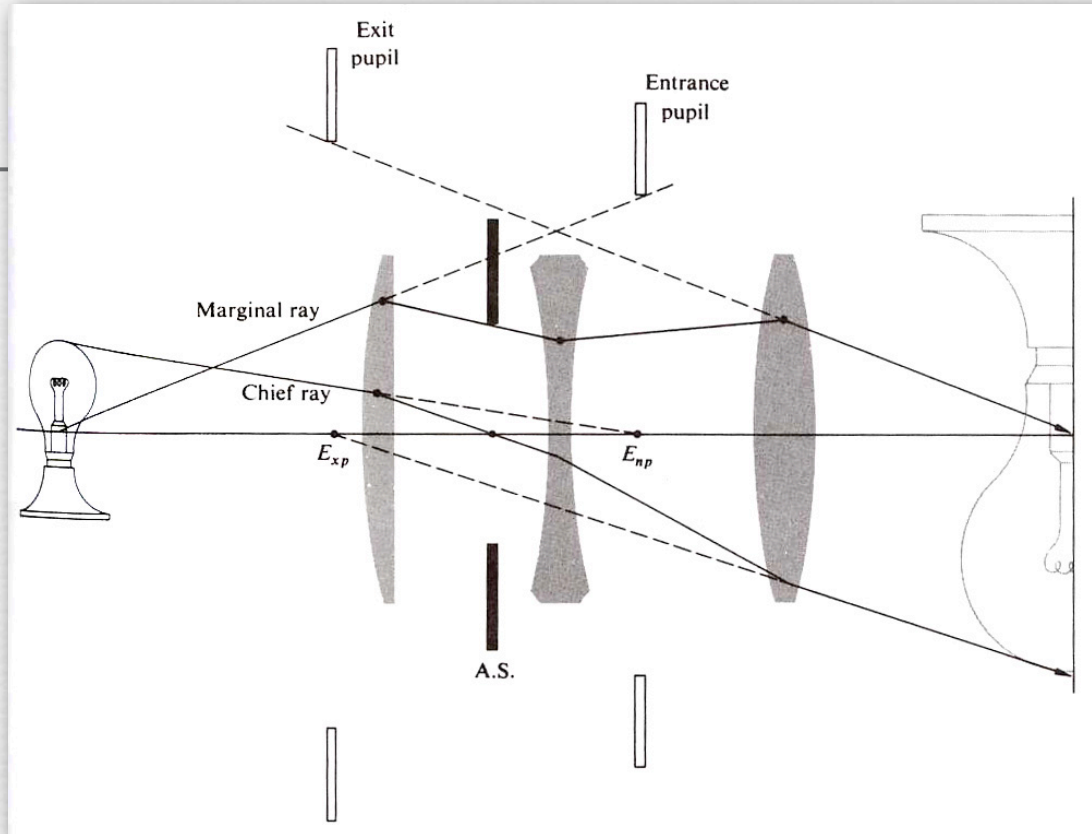
Stops



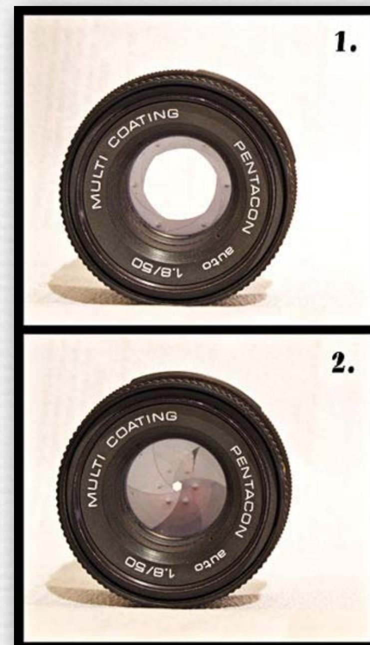
(Hecht)

- in photographic lenses, the *aperture stop* (A.S.) is typically in the middle of the lens system
- in a digital camera, the *field stop* (F.S.) is the edge of the sensor; no physical stop is needed

Pupils



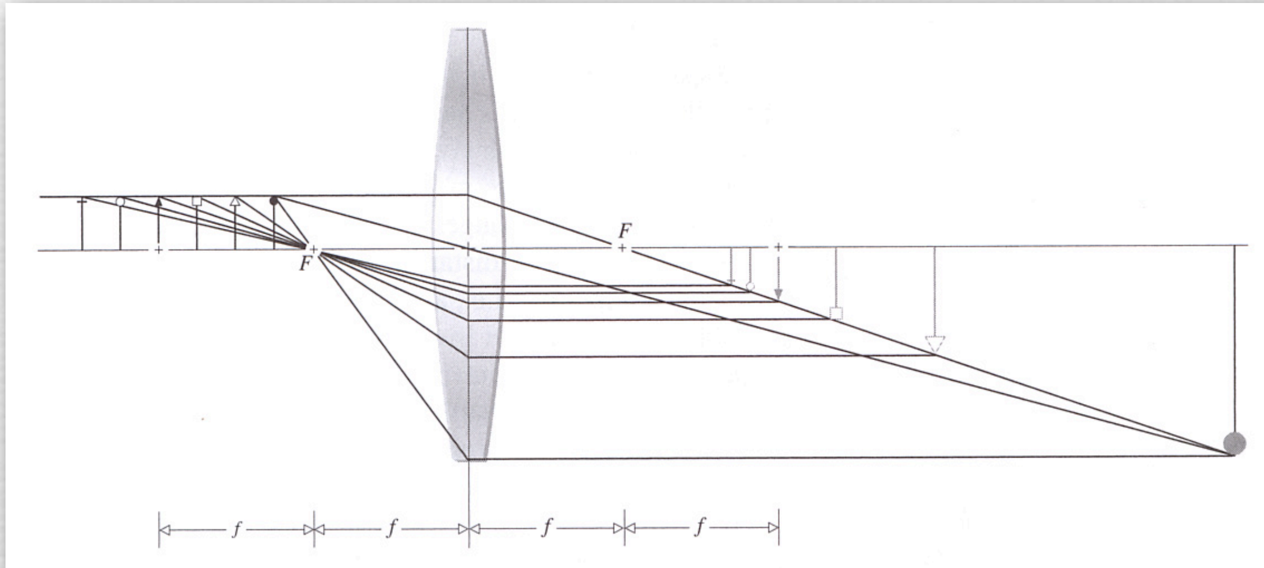
(Hecht)



(wikipedia)

- the *entrance pupil* is the image of the aperture stop as seen from an axial point on the object
- the *exit pupil* is the image of the aperture stop as seen from an axial point on the image plane
- the center of the entrance pupil is the center of perspective
- you can find this point by following two lines of sight

Lenses perform a 3D perspective transform



(Hecht)

(FLASH DEMO)

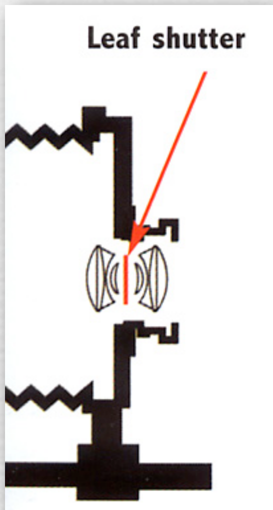
http://graphics.stanford.edu/courses/cs178/applets/thinlens.html

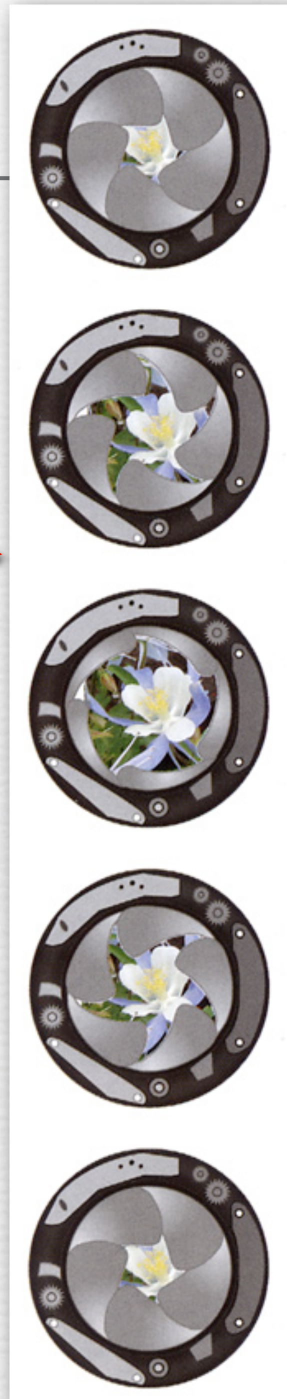
- lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image
- as an object moves linearly (in Z), its image moves non-proportionately (in Z)
- as you move a lens linearly, the in-focus object plane moves non-proportionately
- as you refocus a camera, the image changes size !

Exposure

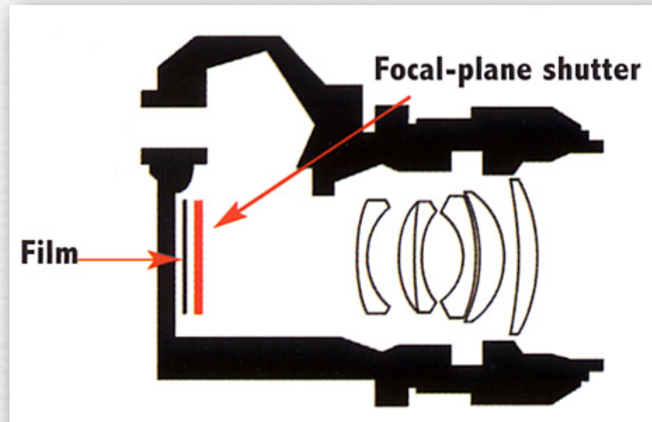
- $H = E \times T$
- exposure = irradiance × time
- irradiance (E)
  - controlled by aperture
- exposure time (T)
  - controlled by shutter

Shutters



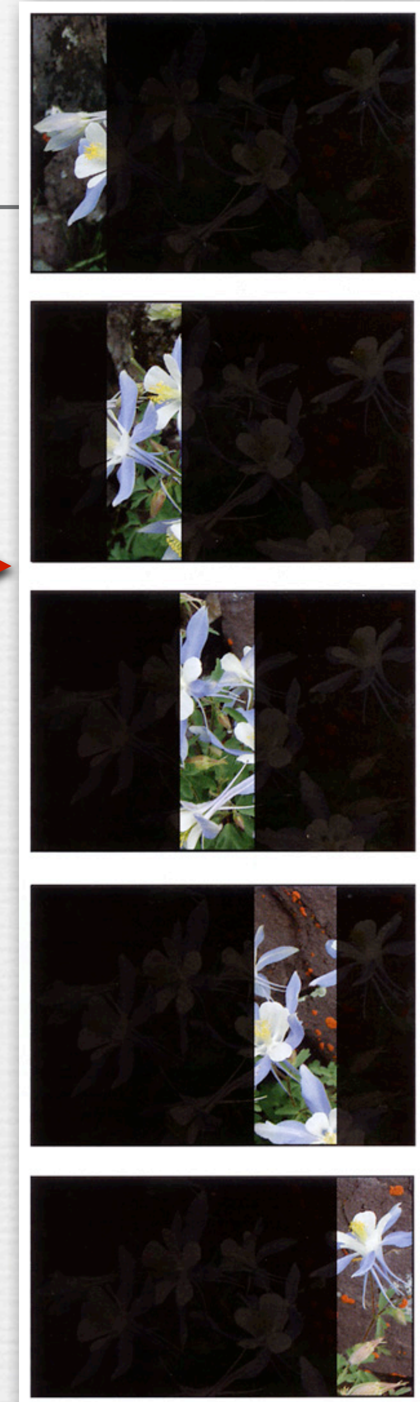


- quiet
- slow
  (max 1/500s)
- need one
  per lens



- loud
- fast
  (max 1/4000)
- distorts motion

(London)



Jacques-Henri Lartigue, Grand Prix (1912)

Shutter speed

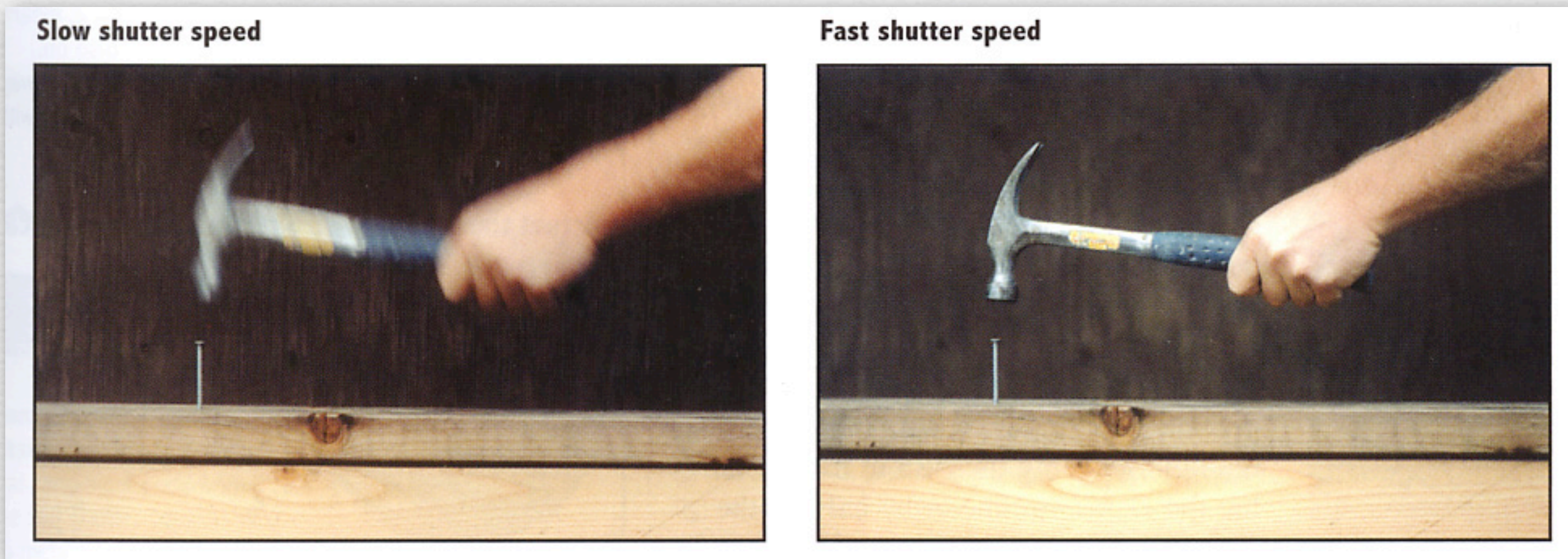
- controls how long the sensor is exposed to light
- linear effect on exposure until sensor saturates

- denoted in fractions of a second:
  - 1/2000, 1/1000,...,1/250, 1/125, 1/60,...,15, 30, B(ulb)

- normal humans can hand-hold down to 1/60 second
  - *rule of thumb:* shortest exposure = 1 / *f*
  - e.g. 1/500 second for a 500mm lens

Main side-effect of shutter speed

- motion blur
- halving shutter speed doubles motion blur



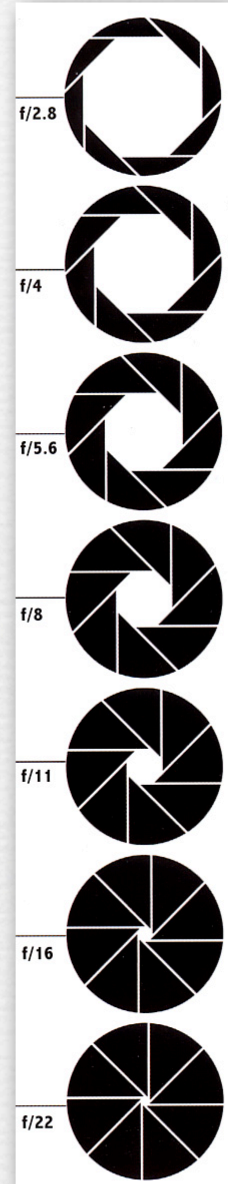
(London)

Aperture

- irradiance on sensor is proportional to
  - square of aperture diameter *A*
  - inverse square of distance to sensor (~ focal length *f*)
- so that aperture values give irradiance regardless of lens, *aperture number N* is defined relative to focal length

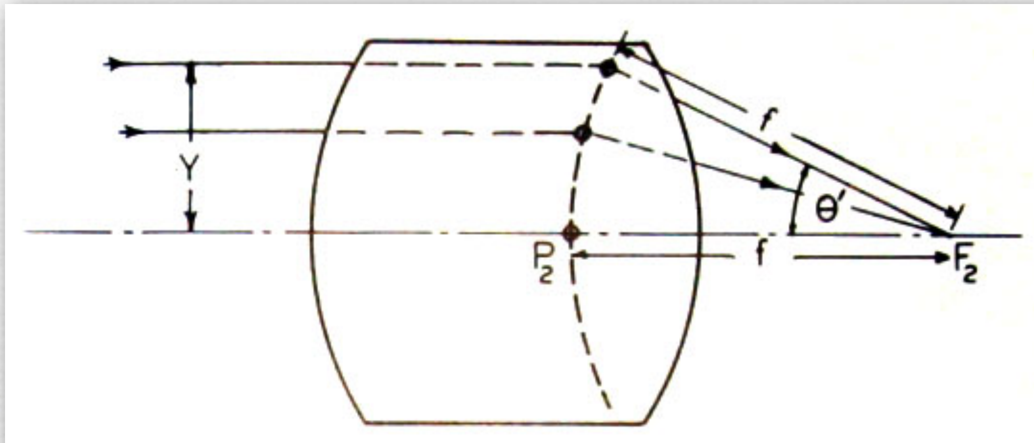
$$N = \frac{f}{A}$$

  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm
  - ∴ low F-number (N) on long zooms require fat lenses
- doubling N reduces A by 2×, hence light by 4×
  - going from f/2.0 to f/4.0 cuts light by 4×
  - to cut light by 2×, increase N by √2



(London)

How low can N be?



(Kingslake)

- principal planes are the paraxial approximation of a spherical "equivalent refracting surface"

$$N = \frac{1}{2\sin\theta'}$$

Canon EOS 50mm f/1.0
(discontinued)

- lowest possible N in air is f/0.5
- lowest N in SLR lenses is f/1.0

Cinematography by candlelight

Stanley Kubrick,
Barry Lyndon,
1975

- Zeiss 50mm f/0.7 Planar lens
  - originally developed for NASA's Apollo missions
  - very shallow depth of field in closeups (small object distance)

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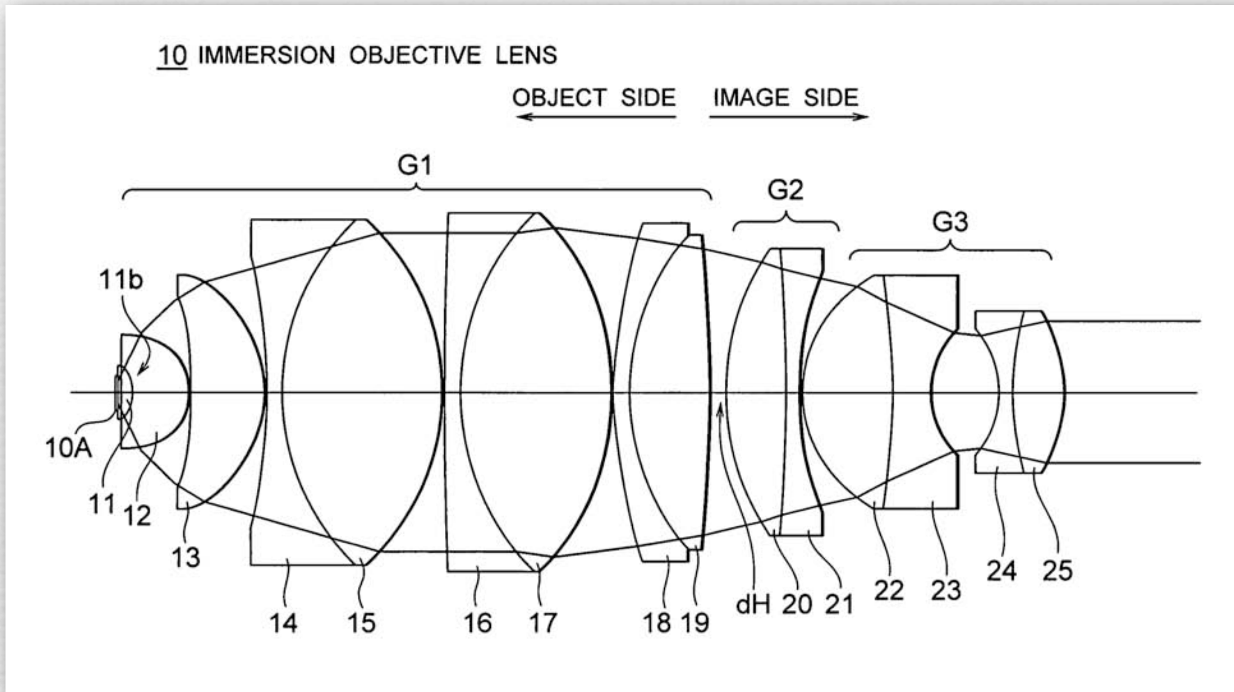
Microscope objectives



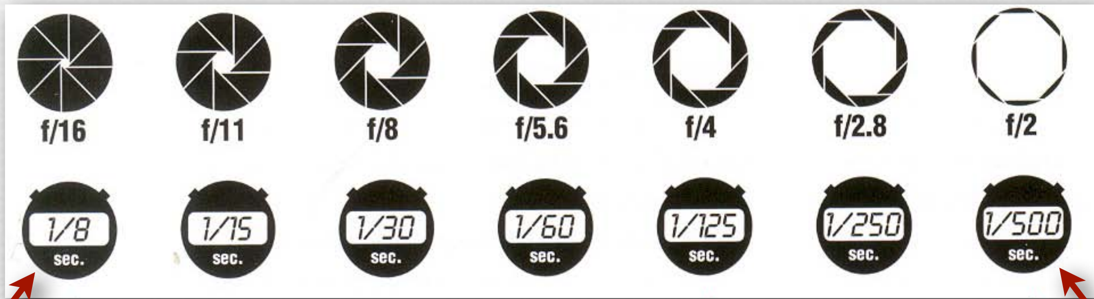
- numerical aperture $NA = n \sin \theta$
- for dry objectives, $N \approx 1/2\ NA$
- so $40\times/\ 0.95NA$ objective = f/0.51 (on object side)!
- $\theta = 71.8°$!

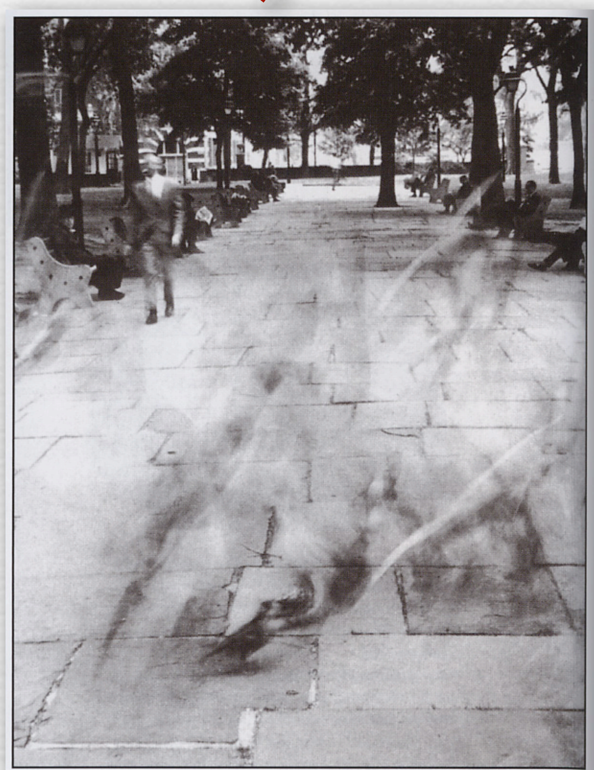
Main side-effect of aperture

- depth of field
- doubling N (two f/stops) doubles depth of field

(London)

Trading off motion blur and depth of field





(London)